Welcome to the fuzzy-verse

We expect the laws of nature that describe the universe to be exact, but what if that isn't true, wonders philosopher **Eddy Keming Chen**
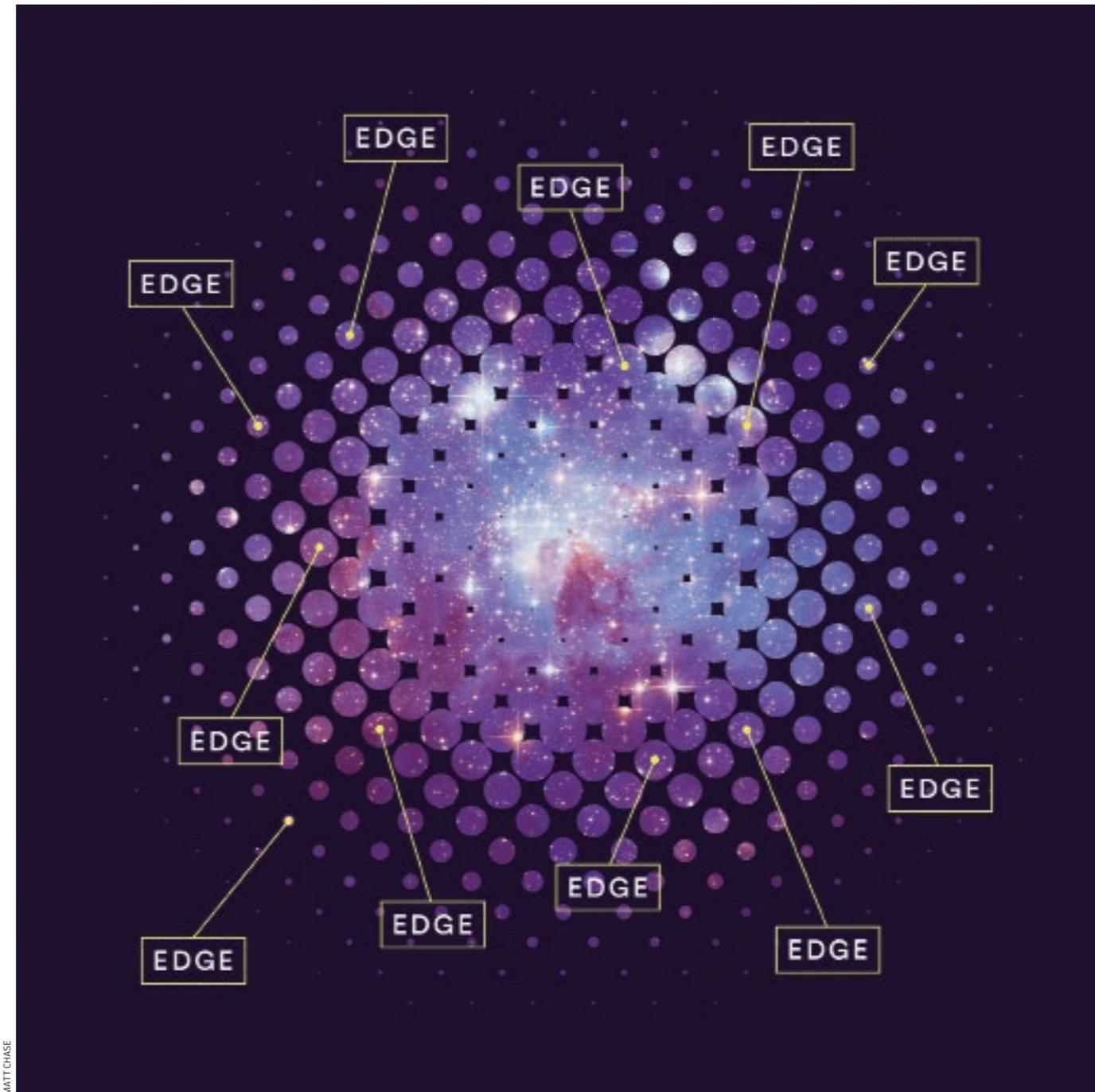
WHAT links a heap of sand, the edge of a cloud and actor Patrick Stewart's baldness? If you are only vaguely grasping what I am getting at, you are on the right track: they are all examples of imprecision in our description of the world. How many grains of sand can you take away from the heap and still call it a heap? Where exactly does the cloud end and the sky begin? How many hairs is Patrick Stewart allowed to have, and of what length, before he is classed as not bald? It is hard – perhaps impossible – to tell.

Such vague concepts, with their messy boundaries and borderline cases, are all around us. Until now, we have tended to assume they represent imperfections in our state of knowledge, our ways of communication or our modes of description. At some level, we think, the world must be precisely defined. Underpinning its workings, in the end, are the laws of physics, which are expressed using cast-iron mathematical equations that admit no vagueness.

I'm not so sure that's the case. I think I have uncovered a fundamental physical law that is itself vague. The implications could be far reaching, potentially casting doubt on the ability of conventional mathematics to provide us with a full description of the universe – but also perhaps opening entirely new avenues to even better physical theories.

Philosophers like me have spent a lot of time thinking about vague terms such as "bald" and "heap". The heap question is known as the sorites paradox, and it was noted as early as the 4th century BC. If a million grains make a heap, then a million minus one grains also make a heap, as do a million minus two grains and so on. Follow that logic, and eventually a single grain also makes a heap. That is absurd. So should we accept that there is a sharp boundary, some number of grains, below which grains don't make a heap? That is hard to swallow, too.

Vagueness is pervasive in natural language, and yet it resists logical analysis. The principle of bivalence that is central to classical logic – every statement is either true or false – seems to fail for vague terms. Imagine Tom, who is a borderline case of "bald". It isn't true that Tom is bald. It isn't false that Tom is bald. Classical bivalent logic is at risk.

Philosophical reflections have identified three types of vagueness. First, there is semantic vagueness. This is just a feature of how we communicate. Perhaps some of the words we use really are so vague that they leave some statements in limbo, neither clearly true nor clearly false.

Second, it may be due to our ignorance of some facts. Even though it may not be clear how to draw a sharp boundary between bald and non-bald, or between heap and non-heap, there may be an objective cut-off that we aren't aware of. This we call epistemic vagueness, and it neatly preserves bivalent logic because there is a true or false answer to any statement, even if we don't know which it is.

Third, vagueness may be due to some genuine indeterminacy in the universe. This is called ontic vagueness. There are some objects we define using natural language, such as a cloud, or Mount Everest, that simply don't have exact boundaries in space or time.

While we can continue to debate what type of vagueness is at play in any one situation, one thing that unites most philosophers is that all of this has little to do with fundamental laws of nature. Vagueness may also appear in some high-level sciences, such as biology, where terms such as "cell", "organism" and "life" are imprecisely defined – a virus of the sort that is exercising us right now seems to be a classic borderline case of a living organism, as a bundle of genetic material that can only replicate inside the cells of another organism. But that vagueness should disappear when we drill down to more fundamental levels of explanation.

Fundamental laws of nature are written in the exact, non-messy language of mathematics. Mathematics, as we currently conceive it, is built around set theory, and a mathematical set is the very definition of being not vague. Something is either

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a member of a set – the set of all odd numbers, say, or all numbers divisible by 11 – or it isn't. Sets are rigidly defined via a notion of equality: if two sets have the same members, they are the same set. Similarly, any mathematical function, topological space or geometrical shape built from sets is precisely defined. It is hard to see how the fundamental laws of physics could be completely and faithfully expressed in these terms if they admitted any vagueness.

On the margins

Take Isaac Newton's universal law of gravitation, for instance, or his second law of motion, force = mass × acceleration (F = ma). Physical laws such as these are arbitrary, in the sense that they are set by nature. From all the different ways the universe could be, these laws pick out a small subset of physically possible versions. They admit no borderline cases: the behaviour of objects within this reality will adhere to these equations exactly, no ifs, no buts.

These laws also have a quality that will become important later on: traceability. Our universe is sensitive to changes in the laws. Any shift in the gravitational constant, *G*, for example, will be felt by massive objects and will change, however slightly or significantly, the motion of planets around stars, the formation of galaxies, the distribution of matter in the cosmos or how a falling vase shatters when it hits the ground. *G*'s exact value leaves a trace on the world. Similarly, if we change F = ma to F = ma$^{1.001}$, it would produce observable physical changes: for a given force acting on a given mass, the resulting acceleration would be less.

The same arguments hold with all other fundamental equations of physics, for example Erwin Schrödinger's equation that defines the evolution of a quantum system, or Albert Einstein's field equations of general relativity that determine the development of the universe at large.

Did I say all? I meant not quite all. There seems to be one essential element of fundamental physics that has every

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Where does a cloud end and the sky begin? Vagueness might be an intrinsic property of how the universe works

> "The Past Hypothesis purports to be a fundamental law, and yet it is screamingly vague"

right to be considered a law, but doesn't fit into this pattern.

Its origin lies in the puzzling observation that while the fundamental equations of physics are all time-symmetric, meaning they work equally well backwards as forwards, the world around us is distinctly time-asymmetric and irreversible. An arrow of time exists, a fact often encapsulated in the second law of thermodynamics, which puts limits on the sort of processes that can occur in reality. Ice cubes melt when placed in a drink to cool it, for example, but don't spontaneously form in it.

Explaining why leads to an influential proposal known as the Past Hypothesis. It says that the universe had a very special starting condition: it was initially in a state of low entropy, one with a high degree of order. This is about as fundamental a law about how the universe works as we have – and yet it is screamingly vague. In its weakest version, it simply says that the initial state of the universe has low entropy. How low is low?

I call this potential vagueness in a fundamental physical law "nomic vagueness".

It seems distinct from the three other types of vagueness, and may be more basic. But let's take a closer look at what it consists of in the case of the Past Hypothesis.

First, its vagueness can be specified in a more precise way. We can characterise the initial state of the universe in terms of macroscopic variables such as temperature, volume, pressure and entropy, in accordance with astrophysical data. However, in classical statistical mechanics, this macrostate corresponds to any number of microstates of individual particles with different positions and velocities. Many different microstates look essentially the same to us as we measure the macrostate. Which microstates correspond to which macrostate is only vaguely defined. There are always going to be borderline cases where a particular configuration of particles might amount to an initial state of that particular temperature, say – or might not.

But what if we just stipulate the exact boundaries of the macrostate by saying the initial state of the universe corresponds to this set of possible microstates and no others? Let us call this the Strong Past Hypothesis. It means that any vagueness about the universe's initial state is due to our inexact knowledge of its macrostate. This is then similar to the epistemic vagueness we discussed earlier. So, nothing to see here?

Untraceable laws

The problem is that this Strong Past Hypothesis is arbitrary, and not just in the way other laws or constants are arbitrary. It is untraceably arbitrary: whereas changing the value of the gravitational constant makes a difference to what the universe is like, there are infinitely many ways of wiggling the boundary of the initial macrostate that make no difference to what the universe is like or even the probabilities of events within it.

This leaves us on the horns of a dilemma: we either embrace nomic vagueness or nomic untraceability. That is to some extent a matter of taste, but I suggest we should avoid untraceability. Observations of the universe

often can uncover the nature of traceable laws. By contrast, untraceable laws can't be pinned down by facts. We can't do science to determine what they are; there is a gap between untraceable laws and the world.

A resolution to this dilemma might still come from within physics, and from a rather surprising quarter – quantum theory.

At first glance, this would seem to be the last place to look to banish vagueness from physical laws. Quantum objects such as particles are described by "wave functions" that have no definite locations in space or other exactly defined properties. Besides reality thus apparently becoming riddled with ontic vagueness, the very process of measurement that resolves this vagueness, "collapsing" quantum wave functions into exact states, is itself painfully vaguely defined. In the words of physicist John Stewart Bell: "What exactly qualifies some physical systems to play the role of 'measurer'? Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system... with a PhD?"

Vagueness is indeed a feature of orthodox quantum theory – but other, competing interpretations are also available. In the many-worlds interpretation, when we probe a quantum system, the universe divides according to the possibilities we might see. There is no vagueness at the fundamental level in this depiction: the fundamental stuff is always exactly defined, and the dynamical laws are exactly specified. Meanwhile, in Bohmian mechanics, also known as pilot-wave theory, a single universe evolves deterministically at all times in accordance with exact mathematical equations. And in "spontaneous collapse" theories, wave-function collapse is just a random and spontaneous feature of the universe's dynamical laws, banishing any vague or mystical special role for the measurer.

How might this help with the Past Hypothesis? The details are complex, but it amounts to the fact that, unlike

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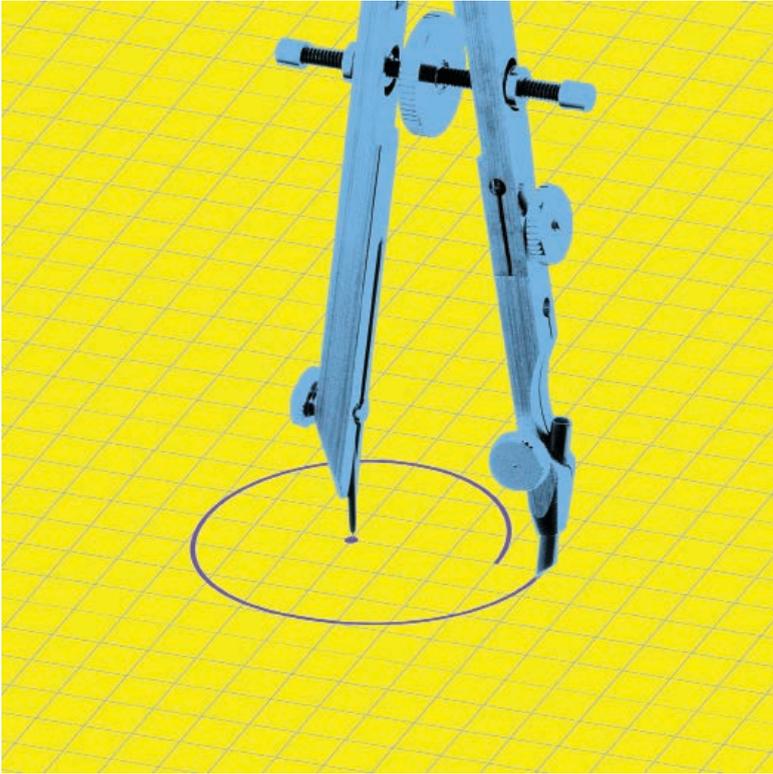
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classical mechanics, quantum mechanics allows us to connect the initial microstate and macrostate of the universe in an exact, traceable way. Traditionally, the initial quantum state of the universe is described by a wave function, and the Past Hypothesis restricts the possible wave functions to a small subset compatible with a low-entropy macrostate. Work I have done shows that we can specify the initial state of the universe as something equivalent to a sum over all these possible wave functions.

Fundamentally vague?

In pilot-wave theories and many-worlds theories, the form of the resulting "initial density matrix" will influence how things evolve subsequently; in spontaneous collapse theories, it will determine how collapses randomly happen. Changes to the initial density matrix will typically alter what the universe is like, just as is the case with other dynamical constants and laws. Thus quantum theory helps us to preserve both nomic exactness and traceability.

But there is no guarantee that such a solution is possible: we are far from achieving a quantum description of the beginning of the universe, and it may well be that a final theory of the nature of reality may not be fully quantum. If so, we again find ourselves on the horns of our dilemma, forced to admit nomic vagueness – and the consequences could be profound, not least for our ability to use mathematics to describe the universe. Any way of capturing a fundamental, yet vague, law such as the Past Hypothesis using traditional mathematics based on

set theory will miss out something, or will impose too much sharpness somewhere.

This is perhaps an opportunity to think beyond classical mathematics for describing the universe. Other foundations besides set theory do exist for mathematics. Category theory, for example, focuses not on which mathematical objects are in which set, but on the abstract relationships between objects. Then there is homotopy type theory, which relaxes the notion of equality between objects central to set theory and defines objects in terms of paths between points in an abstract space. Either approach might provide a better language for capturing all physical laws, offering more flexibility in dealing with vagueness. Equally, it is possible that no mathematics can deal with it.

What about future laws of physics? That is a big unknown. But if nomic vagueness is possible, perhaps we don't have to restrict ourselves to formulating laws that can only be stated in precise mathematics. For example, the physicists Abhay Ashtekar and Brajesh Gupt have recently done some work on loop quantum gravity, one promising approach to unifying quantum theory with Einstein's theory of gravity, general relativity. Their proposal of an initial condition for the universe could be an instance of nomic vagueness, because of a vague boundary of the "Planck regime", the earliest epoch of the universe when the quantum effects of gravity dominated all other forces. It is one hint that a final theory of physics might not be entirely mathematically expressible.

Mathematics will still remain extremely useful. But if there is nomic vagueness, it may never completely capture the objective order of the universe. It may turn out that vagueness runs far deeper than defining the number of grains of sand in a heap or of the hairs on a bald man's head. ∎

"Mathematics may never completely capture the objective order of the universe"

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