Interpreting Supersymmetry

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The very basics

- Supersymmetry (SUSY) is a conjectured symmetry of string theory and beyond-the-standard model quantum field theories (QFTs)
- SUSY transformations relate boson and fermion states

Why interpret SUSY now?

- Recent experimental tests have come up dry.

Why interpret SUSY now?

Physics-wise:

- SUSY can appear at any scale
- String theory probably can't work without it
- It still might solve the hierarchy problem (fine-tuning of the forces) and/or make grand unification possible
- It's not that hard!

Why interpret SUSY now?

Philosophy-wise:

- It involves a new example of explanation by unification
- The most natural formalism posits a strange higher-dimensional geometry (superspace)

Three interpretive questions

1. Is superspace *spacetime*?
2. Should we be substantivalists about superspace?
3. Is superspace required to unify bosons and fermions?

A toy model: The supersymmetric oscillator

- The analogy between harmonic oscillators and quantum fields
- A SHO is a free "QFT" on one-dimensional spacetime
- Quanta of excitation = particles
- Energy of one mode = particle mass
- "Position" operator $x(t)$ is the analogue of field operators

A bosonic oscillator

Raising/creation operator $a^\dagger$, lowering/annihilation operator a.

$$a^\dagger|0\rangle = |1\rangle = a|2\rangle$$

etc

$$[a, a] = [a^\dagger, a^\dagger] = 0$$

$$[a, a^\dagger] = 1$$

Number operator $N_B = a^\dagger a$
Scalar field/"position" operator $\phi(t)$
Hamiltonian $H = m_B N_B$

A fermionic oscillator

Raising/creation operator $c^\dagger$, lowering/annihilation operator c.

$$c^\dagger|0\rangle = |1\rangle, c|1\rangle = |0\rangle$$

$$\{c, c\} = \{c^\dagger, c^\dagger\} = 0$$

$$\{c, c^\dagger\} = 1$$

so

$$c^\dagger|1\rangle = 0$$

(Pauli exclusion principle!)
Number operator $N_F = c^\dagger c$
Spinor-valued Dirac field/"position" $\psi(t)$
Hamiltonian $H = m_F N_F$

The supersymmetric oscillator

- Composite system with a bosonic oscillator and a fermionic oscillator

$$H = H_B + H_F$$

- Stipulate equal masses
- Then replacing one fermion with a boson or vice versa doesn't change the energy

$Q = \sqrt{m} a^\dagger c$ and $Q^\dagger = \sqrt{m} c^\dagger a$ do this

$$[Q, H] = [Q^\dagger, H] = 0$$

Supercharges

$$[Q, H] = [Q^\dagger, H] = 0$$

The generators of a symmetry–supersymmetry!

The Q's are weirder than they seem

- They generate a *supergroup* rather than a Lie group
- They multiply like Grassmann anticommuting numbers
- So $QQ = Q^\dagger Q^\dagger = 0$–no polynomials of order > 1
- This means the Qs are both the generators and the symmetry operators

A connection with spacetime

Supergroups are characterized by anticommutation relations of the generators:

$$\{Q, Q^\dagger\} = \sqrt{m}a^\dagger c\sqrt{m}c^\dagger a + \sqrt{m}c^\dagger a\sqrt{m}a^\dagger c$$

$$= m(a^\dagger a + c^\dagger c)$$

$$= m(N_B + N_F) = H$$

So the Hamiltonian–generator of time translations–is one of the generators of the supergroup!

Superspace formalism

- Add two conjugate Grassmann-valued coordinates θ and $\bar{\theta}$ to spacetime
- The two oscillators are represented jointly by a *superfield:*

$$\Phi(t, \theta, \bar{\theta}) = \phi(t) + \bar{\theta}\psi(t) + \bar{\psi}(t)\theta + F(t)\bar{\theta}\theta$$

- $Q, Q^\dagger$ implement translations in time and $\theta, \bar{\theta}$

Superspace formalism

If superspace is understood as spacetime, SUSY is a spacetime symmetry

Is superspace spacetime?

Menon: Yes, it is *theoretical spacetime* (but not *operational spacetime*)

- "Spacetime" is a functional concept (Knox)
- Spacetime is the structure necessary to define inertial frames (Knox)
- The external dynamical symmetries match the spacetime symmetries (Earman)
- Superspace satisfies these criteria.

Reply to Menon

- "(Theoretical) Spacetime" seems like a cluster concept
- So no necessary and sufficient conditions
- The extra superspace dimensions satisfy Menon's conditions, but may fail others

Further criteria for the spacetime concept

- *Fundamentality* as a criterion?
- This means the substantivalism question is prior to Menon's question
- Minkowski spacetime defines "interactive distance" for forces

A borderline case?

Superspace substantivalism?

Substantivalism: Spacetime is at least as fundamental as matter (North)

- For a non-fundamental theory: Among the most fundamental structures in its domain?
- Which is more fundamental: Superspace or Minkowksi spacetime?

Superspace as surplus structure?

- Superspace and Minkowksi space versions of a SUSY QFT have the same algebra of observables
- So who needs the extra superspace structure?
- Is superspace structure explanatorily superfluous?

An argument for superspace

- Premise: Fundamental geometric structure can explain symmetries
- *Contra* Brown, dynamical symmetries are governed by fundamental geometry
- Then fundamental superspace explains why SUSY is a symmetry

Is superspace needed to unify bosons and fermions?

"If nature is supersymmetric, then force and matter appear in the same multiplets. But is this really unification? The answer appears to hinge on how realistically we can (or should) interpret superspace." (Weingard, "A Philosopher Looks at String Theory")

- Fermions and bosons can only be unified if SUSY is a spacetime symmetry

Weingard's argument

- Spin-up and spin-down electrons are physically equivalent because they're related by rotations
- The same reasoning doesn't carry over to isospin transformations
- Isospin space is "just a mathematical space"
- So the symmetry just means "the particles can be transformed into each other"

Against Weingard

- Isospin is not an exact symmetry!
- Phase transformations in QM are also rotations in a “mathematical space”
- Does Weingard’s perspective make sense applied to gauge symmetries?