Realism about the Wave Function

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Abstract

A century after the discovery of quantum mechanics, the meaning of quantum mechanics still remains elusive. It is in large part due to the puzzling nature of the wave function, the central object in quantum mechanics. The wave function is defined on a high-dimensional space, has values in the complex plane, and is only unique up to an overall phase. If we are realists about quantum mechanics, then how should we understand the wave function? What does it represent? What is its physical meaning? Answering these questions would improve our understanding of what it means to be a realist about quantum mechanics. In this survey article, I review and compare several realist interpretations of the wave function. They fall into three categories: the ontological interpretations, the nomological interpretations, and the *sui generis* interpretation. For simplicity, I will focus on non-relativistic quantum mechanics.

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1 Introduction

Quantum mechanics is one of the most successful physical theories to date. Not only has it been confirmed through a wide range of observations and experiments, but it also has led to technological advances of a breathtaking scale. From electronics, optics, computing, to nanoscience, applications of quantum mechanics are ubiquitous in our lives.

As much as it has given rise to technological innovations, the meaning of quantum mechanics remains elusive. Many curious features of quantum mechanics, such as entanglement, non-locality, and randomness, are taken to be *prima facie* challenges for a clear understanding of quantum mechanics. These puzzles are related to the *wave function*, a central piece of mathematical device that is crucial for the predictions of quantum mechanics. Understanding the meaning of quantum mechanics seems to require a good understanding of the meaning of the wave function.

What does the wave function represent? That is the main concern of this survey article. However, it is not an easy question to answer, partly due to the fact that it does not look like anything familiar. It lives on a high-dimensional space, assigns complex values, and is unique only up to an overall phase. Nevertheless, we have devised many ways of using wave functions in making predictions and explaining phenomena. We know how to associate a wave function with a system of electrons, for example. We use the wave function to calculate the probabilities of the microscopic and macroscopic behaviors of the system, including the hydrogen atom, the double slit experiment, and the Stern-Gerlach experiment. The wave function is indispensable in making predictions. However, the predictions are probabilistic. Roughly speaking, there are three main views about the wave function:

Instrumentalism: The wave function is merely an instrument for making empirically adequate predictions.

Epistemicism: The wave function merely represents the observer's uncertainty of the physical situation.[1]

Realism: The wave function represents something objective and mind-independent.

In this article, I focus on the realist interpretations of the wave function. They seem to be the most interesting and promising ways of understanding quantum mechanics.

[1]The recently published theorem of Pusey et al. (2012) shows that a certain class of epistemic interpretations of the wave function are incompatible with the empirical facts.

Let me make four remarks. First, the meaning of the wave function is related to solutions to the quantum measurement problem. Hence, we will start in §2 with an introduction to this topic, along with some mathematical preliminaries. Second, I left the definition of *realism* open-ended. This is because we will consider proposals that provide specific versions of realism about the wave function. The proposals are grouped into three categories: the ontological interpretations (§3), the nomological interpretations (§4), and the *sui generis* interpretation (§5). Third, because of the prevalence of quantum entanglement, "the wave function" should be understood to refer to the wave function of the universe, or the universal wave function. The wave functions of the subsystems are thought to be derivative of the universal one. Fourth, for the sake of simplicity, I will focus on non-relativistic versions of quantum mechanics.[2]

The issues taken up here are continuous with the general question about how to interpret physical theories. They offer concrete case studies for scientific realism, and they might be useful for both philosophers of science and metaphysicians.

2 Background

In this section, we will review the mathematics of the wave function and its connection to the probabilistic predictions. We will then consider the quantum measurement problem and three realist theories that solve it. The upshot is that the wave function occupies a central place in their descriptions of physical reality.

2.1 The Wave Function

It will be useful to have a brief review of classical mechanics. To describe a classical mechanical system of N particles, we can specify the position $\boldsymbol{q}$ and momentum $\boldsymbol{p}$ of each particle in physical space (represented by $\mathbb{R}^3$). We can represent the classical state of an N-particle system in terms of $6N$ numbers, $3N$ for positions and $3N$ for momenta. The classical state can also be represented as a point in an abstract state space called the *phase space* $\mathbb{R}^{6N}$. Once we specify the forces (or interactions) among the particles, they evolve deterministically, by the Hamiltonian equations of motion:

$$\frac{\partial \boldsymbol{q}_i}{\partial t} = \frac{\partial H}{\partial \boldsymbol{p}_i}, \frac{\partial \boldsymbol{p}_i}{\partial t} = -\frac{\partial H}{\partial \boldsymbol{q}_i}, \tag{1}$$

where H stands for the Hamiltonian function on the phase space, which is a shorthand notation for the classical interactions such as Newtonian gravitational potential and Coulomb electric potential. The Hamiltonian equations are differential equations, and the changes in the particles are obtained from taking suitable derivatives of H. In this sense, H is the generator of motion. For every point in the phase space, H generates a curve starting from that point. In other words, for every initial condition of the N particle system, H determines the future trajectories of the particles.

[2]For complications that arise in the relativistic theories, see Myrvold (2015).

Now let us introduce the quantum mechanical way of describing a system of N "particles." Instead of describing it in terms of the positions and momenta of N particles, we use a wave function for the system. The wave function represents the *quantum state* of the system. In the position representation, the wave function, denoted by $\psi(q)$, is a particular kind of function from the configuration space $\mathbb{R}^{3N}$ to complex numbers $\mathbb{C}$. Let us elaborate on this definition:

- Domain: the domain of the wave function ψ is $\mathbb{R}^{3N}$, or N copies of physical space $\mathbb{R}^3$. Hence, we can write ψ as $\psi(\boldsymbol{q_1}, ..., \boldsymbol{q_N})$, where q_i is a point in physical space. The reason for using N copies of physical space is because we are dealing with a system of N "particles," and in general they will be "entangled." If we regard the universe as a quantum system, N is a very large number, of the order of 10^{80}. Each point in $\mathbb{R}^{3N}$ represents a configuration of N "particles" in $\mathbb{R}^3$. Hence, $\mathbb{R}^{3N}$ is called the *configuration space*.[3]

- Range: the range of the wave function ψ, in the simplest case, is the field of complex numbers $\mathbb{C}$. A complex number has the form $a + bi$, where $i = \sqrt{-1}$; in polar form, it is $Re^{i\theta}$, where R is the amplitude and θ is the phase. (If we include spinorial degrees of freedom, the range is $\mathbb{C}^k$. We set spins aside in this paper.)

- Restrictions: the wave function is a particular kind of functions from $\mathbb{R}^{3N}$ to $\mathbb{C}$; it has to be "square-integrable." That is, if we take the square of the amplitude of the wave function value at every point, and integrate over the entire configuration space, we will get a finite value. This is to ensure that we can normalize the squared value of the wave function to 1 so that it has meaningful connections to probabilities. To ensure that we can take suitable derivatives on the wave function, we often also require the wave functions to be sufficiently *smooth*.

- Abstract state space: each wave function ψ is a member of $L^2(\mathbb{R}^{3N}, \mathbb{C})$ the set of square-integrable functions from the configuration space to complex numbers. $L^2(\mathbb{R}^{3N}, \mathbb{C})$ is an instance of a special vector space called the Hilbert space. Each wave function is a vector in that Hilbert space.

In classical mechanics, the state of a system is represented by the positions and momenta of all the N particles (a point in phase space) that changes deterministically according to (1). If the wave function represents the quantum state of a system at a time, how does it change over time? It obeys another differential equation called the *Schrödinger equation*:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi, \tag{2}$$

[3]This is the ordered configuration space, in which a permutation of the particle labels creates a different configuration. If the particles are indistinguishable, then it is more natural to use the *unordered configuration space*, ${}^N\mathbb{R}^3$. This has implications for the nature of the wave function. See Chen (2017b) and the references therein.

where i is the complex number $\sqrt{-1}$, $\hbar$ is the Planck constant divided by 2π, and $\hat{H}$ is the Hamiltonian operator that encodes the energy and fundamental interactions in nature. It is also deterministic: given any vector in the Hilbert space, the Schrödinger equation (2) produces a determinate curve in the Hilbert space. Another important feature of the (2) is that it is linear: if ψ_1 and ψ_2 are solutions to the equation, then their linear combinations are also solutions to the equation. A striking consequence of linearity is that, in the Schrödinger's cat thought experiment, the cat can be in a superposition of the alive state and the dead state.

$$\psi_{cat} = \frac{1}{\sqrt{2}}\psi_{alive} + \frac{1}{\sqrt{2}}\psi_{dead}, \tag{3}$$

A cat in this quantum state is not alive, and it is not dead. The linear Schrödinger equation (2) does not determine a unique outcome of experiment. To resolve this, textbook quantum mechanics says that the Schrödinger equation is to be supplemented with additional collapse postulates. Whenever we open the box and "observe" the cat, the system will collapse into one of the two states: the cat is either alive or dead. An important role of the wave function is determining the probabilities of experimental outcomes, which are taken to be the results of wave function collapses. For example, the probability of finding the system in any set of configurations is given by the Born rule:

$$P(q \in A) = \int_A |\psi(q)|^2 dq, \tag{4}$$

where A is a set of points in the configuration space, $|\psi(q)|^2$ is the squared amplitude of the wave function, and dq is the Lebesgue measure on $\mathbb{R}^{3N}$. In the cat example, the probability of finding the cat to be alive is equal to $\frac{1}{2}$, since $\int |\frac{1}{\sqrt{2}}\psi_{alive}|^2 + 0 = \frac{1}{2}$. The Born rule has the consequence that wave functions that differ only by an *overall phase* (multiplied by a complex number $e^{i\theta}$, where $\theta \in [0, 2\pi]$) will give rise to the same observable phenomena ($|\psi|^2 = |e^{i\theta}\psi|^2$). That is called the *overall phase symmetry*, which motivates the common view that two wave functions that differ by an overall phase represent the same quantum state.

2.2 Quantum Measurement Problem

Notwithstanding the empirical success of quantum mechanics, the collapse postulates seem out of place for a fundamental theory of the world. If the wave function (of the system and the measurement device) obeys the Schrödinger equation, how can it also obey the collapse postulates that contradict the linearity of the Schrödinger equation? But if the wave function does not collapse, how can we obtain unique outcomes of experiments? In short, we have the *quantum measurement problem*:

(P1) The wave function is the complete description of the physical system.

(P2) The wave function always evolves by the Schrödinger equation.

(P3) Every experiment has a unique outcome.

Each of the three propositions is plausible. However, together they lead to a contradiction. To see the contradiction, let us apply them to Schrödinger's cat thought experiment. If P1 is true, then the system is completely described by (3). If P2 is true, then the wave function never collapses into one of the definite states. If P3 is true, the cat is nonetheless in one of the definite states—either alive or dead.[4]

Since P1—P3 are inconsistent, at least one of them is false. Rejecting P1 and P2 would require us to develop alternative theories of quantum mechanics. Rejecting P3 would lead to major revisions about our assumptions of the world. There are three main "interpretations " of quantum mechanics that carry out such strategies. They all contain significant revisions of quantum mechanics, so we should call them realist theories of quantum mechanics instead of interpretations.

First, the de Broglie-Bohm theory, or Bohmian mechanics (BM), rejects P1. According to BM, the wave function is not the complete description of the physical system. There are actual particles with precise positions in physical space. The wave function still obeys the Schrödinger equation. But it also determines the velocity of the particles according to the *guidance equation*.[5] In the cat example, the cat is made out of particles in physical space. There is always a determinate configuration of particles, so the cat is either alive or dead. The probabilities of quantum mechanics become epistemic uncertainties over initial particle configurations.[6]

Second, the Ghirardi-Rimini-Weber theories of spontaneous collapse (GRW) reject P2. According to GRW theories, the wave function Ψ_t does not always obey the Schrödinger equation. It undergoes spontaneous collapses with a fixed rate per particle per unit time. In the cat experiment, given the vast number of particles in the system, it will quickly collapse into a determinate state in which the cat is either alive or dead. Collapses are represented by Gaussian functions with a fixed width in physical space. Due to entanglement, this has the effect that the entire wave function will collapse into a definite state. On the macroscopic scale, the collapse will give rise to Born rule probabilities. Each version of GRW postulates specific values for the collapse rate and the Gaussian width. Moreover, there can be additional variable representing ontology in physical space. GRWm adds a mass-density ontology that specifies the amount of mass in physical space by a real-valued function $m(x,t)$

[4]For a more thorough discussion about the quantum measurement problem, see Myrvold (2017) and Bell (1990).

[5]The particles move according to the guidance equation:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\nabla_i \psi}{\psi}, \tag{5}$$

where Q_i and m_i are the position and mass of particle i, Im means taking the imaginary part, and ∇_i means taking the gradient with respect to the i-th particle. The particles are initially distributed according to the Born rule, and their distribution will always agree with the Born rule because of the mathematical properties of the Schrödinger equation and the guidance equation.

[6]For a survey of BM, see Goldstein (2017); for the original paper, see Bohm (1952); for a modern version, see Dürr et al. (1992).

with, where x, t is a space-time point.[7] In contrast, GRWf adds a flash ontology that postulates the existence of space-time events at the center of the Gaussian function. It can be represented as a function $F(x,t)$ with $x \in \mathbb{R}^3$ and $F(x,t) = 1$ if (x,t) is the center of some GRW collapse and 0 otherwise.[8]

Third, the Everettian quantum mechanics (EQM) rejects P3. According to EQM, there is no need to ensure that there is a unique outcome in the cat experiment. There simply are two branches of the wave function, one in which the cat is alive and the other in which the cat is dead. Both branches co-exist. Because of a property called *decoherence*, the branches do not interfere much with each other. The branches of the wave function are emergent worlds, the wave function is the complete description of the world, and it always obeys the Schrödinger equation. Similarly to GRWm, we can devise a version of EQM with a mass-density ontology. This is first proposed by Allori et al. (2010) and is called Sm. A challenge for any version of EQM is how to make sense of probability in a world in which every possible outcome of every quantum experiment happens with certainty.[9]

The upshot is that the wave function figures prominently in all three realist quantum theories. In BM, although the wave function is not the complete description of the system, it is still part of the description. Moreover, the wave function guides particle motion. In GRW, the wave function collapses and gives rise to unique outcomes of experiments. In EQM, the wave function never collapses but gives rise to emergent parallel worlds. For quantum theories with additional ontology, such as BM, GRWm, GRWf, and Sm, the wave function is also tied to the dynamics of the additional ontology. But their relationship is different in these theories. Bohmian particles have independent dynamics: even if the wave function were not to change, Bohmian particles would still move in a non-trivial fashion. That is not the case in GRWm, GRWf, and Sm. Had there been no change to the wave function, the additional ontology would not change either. It is in this sense that the dynamics of such additional ontologies are not independent of the dynamics of the wave function.

3 Ontological Interpretations

In this section, I review four *ontological* interpretations of the wave function. However, the label "ontological" could be misleading. These four interpretations share the feature that the wave function is interpreted as part of the fundamental *material ontology*, on a par with particles, fields, space-time events or properties, which are the kind of microscopic things that make up macroscopic objects such as tables and

[7]The mass-density function is defined from the wave function:

$$m(x,t) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} d^3x_1 ... d^3x_N \delta^3(x_i - x) |\Psi_t(x_1, ..., x_N)|^2 \quad (6)$$

[8]For a survey of GRW, see Ghirardi (2018); for the original paper, see Ghirardi et al. (1986); Bell (2004), Ch 22, contains a clear presentation of the theory.

[9]For a survey of EQM, see Vaidman (2018); for the original paper, see Everett III (1957); for an updated book-length development of the theory, see Wallace (2012).

chairs. In §4 and §5, we will review the nomological interpretations and the *sui generis* interpretation, which are compatible with the position that the wave function is part of the ontology but just not in the same ontological category as particles or fields.

3.1 A Field on a High-Dimensional Space

According to the first ontological interpretation, the fundamental space is a high-dimensional space, and the wave function is a field in that space. This was introduced by Albert (1996) and developed by Loewer (1996), Ney (2012), and North (2013), among others. Albert calls this view *wave function realism*. However, as we shall see in the later sections, that label is no longer appropriate given the abundance of other approaches that are also realist about the wave function.

This is a counter-intuitive proposal. However, we can start with something familiar and draw analogies with classical physics. In classical field theories such as Maxwellian electrodynamics, electromagnetic fields are fields on the four-dimensional physical space-time. A field on physical space-time can be interpreted as an assignment of monadic properties (field strength and direction) to each point in space-time. Such an assignment is determined by Maxwell's equations and certain boundary conditions.

In a similar way, the wave function can be interpreted as a physical field. However, the wave function cannot be interpreted as a field on physical space, as its domain is the high-dimensional configuration space, represented by $\mathbb{R}^{3N}$. If we take the configuration space to be fundamental, then the wave function can be interpreted as a field that assigns properties to each point in the configuration space. The shape of the field changes over time, according to the Schrödinger equation. On this view, the high-dimensional configuration space is ontologically prior to the physical space(time), and the latter somehow comes out of the fundamental structure.[10]

[10]There are three versions of this view:

- Bohmian version: the fundamental space is represented by $\mathbb{R}^{3N}$. The fundamental ontology consists in a point particle located in that space and a field that assigns properties to points of that space. The field always evolves by the Schrödinger equation. The point particle moves along in the field according to the guidance equation, much like corks move along in flowing river. Here we see a dis-analogy with the classical field. In classical physics, the field and the particles satisfy the action-reaction principle; the fields and the particles can influence each other. In Bohmian mechanics, the wave function interpreted as a field can influence the particle but not vice versa.

- GRW version: the fundamental space is represented by $\mathbb{R}^{3N}$. The fundamental ontology consists in a field that assigns properties to points of that space. The field evolves by the Schrödinger equation most of the time but sometimes collapses by the GRW collapse mechanism.

- Everettian version: the fundamental space is represented by $\mathbb{R}^{3N}$. The fundamental ontology consists in a field that assigns properties to points of that space. The field always evolves by the Schrödinger equation.

The high-dimensional field interpretation of the wave function is incompatible with GRWm, GRWf, or Sm.

The high-dimensional field interpretation prioritizes the structure of the wave function and its dynamics. They are mathematically represented as happenings on the high-dimensional configuration space. Given the analogy with classical fields, the configuration space is interpreted as fundamental such that the wave function is a field on that fundamental space. However, a key challenge to this view is how to explain our experiences, which seem to happen in a three-dimensional space. This is not just a question about recovering the manifest image, but it is also about whether such an interpretation of quantum mechanics can be "empirically coherent," in the sense that if our evidence for quantum mechanics comes from instrument readings in the three-dimensional space, the theory should not undermine such evidence and should explain how the appearance of three-dimensional objects come out the high-dimensional fundamental space.[11]

Albert (1996) suggests that the explanation lies in the dynamics—in the shape of the Hamiltonian operator. Although all the $3N$ dimensions are metaphysically on a par:

$$\{q_1, q_2, q_3, q_4, q_5, q_6, ..., q_{3N-2}, q_{3N-1}, q_{3N}\} \tag{7}$$

the Hamiltonian operator has a term that encodes fundamental interactions takes a particular form:

$$\sum_{0 \leq i < j \leq N} \sum V_{ij}[(q_{3i-2} - q_{3j-2})^2 + (q_{3i-1} - q_{3j-1})^2 + (q_{3i} - q_{3j})^2] \tag{8}$$

The Hamiltonian operator groups the coordinates in the $3N$-dimensional configuration space into triplets, such that there might be emergent objects that have the same functional profile as what we take to be ordinary objects in the 3-dimensional space. This provides reasons to believe that there might be an emergent 3-dimensional physical space. However, Albert's proposed explanation has been challenged by Monton (2002), Lewis (2004), and Chen (2017b). See Emery (2017) for an objection based on conservativeness principles.

Maudlin (2013) has criticized Albert's proposal on the ground that it reifies too much structure. The common view (§2.1) holds that two wave functions that differ by a complex multiplication constant represent the same physical state. But if we interpret the wave function as a field that assigns monadic properties to points in the configuration space, then we would distinguish two wave functions related by a constant, for the numbers assigned to the points are different. This problem can be avoided if we adopt an intrinsic characterization of the wave function, in terms of comparative relations that are invariant under the change by a constant (Chen (2017a)).

3.2 A Multi-field on Physical Space

The high-dimensional field interpretation of the wave function faces difficulties, primarily because it privileges the configuration space over physical space. There

[11] See Barrett (1999) and Barrett (1996).

are many good reasons to take physical space to be ontologically more basic. First, it underlies many important symmetries in physics. Second, it is much easier for a theory to be empirically coherent if it does not undermine the relative fundamentality of physical space(time).

These difficulties are avoided in the second ontological interpretation, according to which the fundamental space is the ordinary physical space(time). On this view, the wave function is not a field in the traditional sense, but a *multi-field* on physical space. (See Forrest (1988), Belot (2012), Chen (2017a,b), Hubert and Romano (2018).) A multi-field is similar to a field. However, unlike fields, multi-fields assign properties not to individual points but to *regions* of points in space. Such regions can be connected or disconnected. The wave function is a function from N copies of $\mathbb{R}^3$ to complex numbers. Instead of thinking it as a field that assigns properties to every point in $\mathbb{R}^{3N}$, we can think of it as a "multi-field" that assigns properties to every region of $\mathbb{R}^3$ that is composed of N points. The multi-field interpretation is a more faithful representation for "indistinguishable particles," since spatial regions understood as mereological fusions are *unordered*. It has the additional advantage of automatically enforcing what is called "permutation invariance": mere permutations of a configuration of N particles do not change the physical state.[12] Similar to the previous interpretation, here we can avoid postulating too much structure by using an intrinsic account of the multi-fields.

3.3 Properties of Physical Systems

The third ontological interpretation, proposed by Wallace and Timpson (2010), affirms the (relative) fundamentality of the physical space(time). On this view, the universe is divided into subsystems that occupy some spatial-temporal regions. Systems can be made out of unions of other subsystems. And the universe is the union of all subsystems. Although not every system has a wave function (because of entanglement), we can still associate to each system a determinate property represented by what is called a *density matrix*. A density matrix will encode all the dynamical variables of the system. Here we should not think of the density matrix as a field or multi-field. Rather, it is thought to be an abstract operator in the Hilbert space.[13] This view was introduced as an alternative to the high-dimensional

[12]Chen (2017b) suggests that the symmetrization postulate is better explained by the low-dimensional interpretation than the high-dimensional interpretation.

[13]For example, if the universe consists in two systems A and B, and if their joint quantum state is this:

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle_A |\beta\rangle_B - |\beta\rangle_A |\alpha\rangle_B) \tag{9}$$

then the density matrix associated with system A (called the *reduced density matrix*) will be:

$$\rho_A = \frac{1}{2}(|\alpha\rangle_A \langle\alpha|_A + |\beta\rangle_A \langle\beta|_A) \tag{10}$$

On this view, neither $|\Psi_{AB}\rangle$ nor ρ_A are understood as functions or fields on some spaces. Rather, they are understood as structures in the abstract Hilbert spaces: $|\Psi_{AB}\rangle$ is a vector in the total Hilbert space $\mathscr{H}_A \otimes \mathscr{H}_B$, and ρ_A is an operator that maps vectors to vectors in the system A's Hilbert space $\mathscr{H}_A$.

field interpretation. It is also an alternative to the low-dimensional multi-field interpretation. However, it is still a version of realism about the wave function, since the universal wave function (or the universal density matrix) is to be found in the ontology—the property of the entire universe.

Wallace and Timpson call this approach *spacetime state realism*. They argue that this approach avoids privileging the position representation of the wave function, and that it has significant advantages in reconciling with relativistic invariance. See Swanson (2018) for some discussions about the relativistic extensions.

A question about spacetime state realism is whether the fundamental ontology contains redundant information. If we help ourselves to a decomposition of the universe into subsystems, and if we have the quantum state of the universe, then we can obtain density matrices of the subsystems by a purely mathematical procedure of tracing out the environmental degrees of freedom. Since they can be derived from the quantum state of the universe, the properties of the subsystem need not be placed in the fundamental ontology. If we get rid of the subsystem properties and only keep the universal property (the universal density matrix), then this approach would be in the same spirit as the low-dimensional multi-field approach in §3.2.

3.4 A Vector in the Hilbert Space

The final ontological interpretation of the wave function takes the abstract Hilbert space even more seriously. Recall that the wave function is represented as a vector in Hilbert space, and the Schrödinger equation can be represented as a equation for vector rotation in Hilbert space. Carroll and Singh (2018) suggest that the Hilbert space is the fundamental space, and the wave function is just a vector in that space. Every goings-on in the world correspond to some particular direction the vector is pointing.

Since the Everettian interpretation of QM is the most natural place for this view, Carroll and Singh (2018) calls this approach *Mad-Dog Everettianism*. In their words, the label is "to emphasize that it is as far as we can imagine taking the program of stripping down quantum mechanics to its most pure, minimal elements."

It is already difficult to recover ordinary objects from the configuration space. It is even more difficult to recover them from the Hilbert space. For one thing, there is no space-time structure in the Hilbert space. The state of the world corresponds to a vector, which is just like every other vector. How can anything familiar, such as space, time, and ordinary objects, come out of a vector in a high-dimensional Hilbert space? Like Albert (1996), Carroll and Singh propose that the answer lies in the structure of the Hamiltonian operator. The Hamiltonian provides a privileged way to decompose the total Hilbert space into smaller spaces, which may explain the emergent structure. This proposal is more speculative than the high-dimensional field interpretation. However, it is in part motivated by the non-fundamentality of space-time in several theories of quantum gravity. As such, it could be a methodologically fruitful project to explore.

4 Nomological Interpretations

According to the ontological interpretations, the wave function is part of the fundamental material ontology of the world, such as particles and fields in classical mechanics. In contrast, the nomological interpretations hold that the wave function is nomological, i.e. on a par with laws of nature. In this section, I survey two kinds of nomological interpretations of the wave function: the strong nomological interpretations and the weak nomological interpretations.

These interpretations are most compelling from a Bohmian point of view. However, they can be adapted for some versions of GRW theories and Everettian theories with additional ontologies.

4.1 Strong Nomological Interpretations

To appreciate the strong nomological interpretations of the wave function, it would be helpful to review the status of the Hamiltonian function in classical mechanics. As mentioned in §2.1, the Hamiltonian equations (1) govern the motion of classical particles in physical space, represented by a curve in the phase space. The Hamiltonian function is the generator of such motion. It is a convenient short hand for the kinetic energy terms and the pair-wise interactions of the particles. We can, if we like, write out H explicitly as a function (of position and momentum) on the right hand sides of the equations. For the Hamiltonian equations to be simple laws of nature, H has to be a simple function. In this sense, we give H a nomological interpretation. Although it is a function on phase space, we do not treat it as part of the ontology.[14]

Let us now consider Bohmian mechanics. In this theory, the guidance equation governs the motion of the Bohmian particles in physical space, represented by a curve in the configuration space. The wave function is the generator of such motion. If the wave function turns out to be a simple function, then we can write out ψ explicitly as a function (of configuration variables) on the right hand side of the equation. Thus, if the wave function turns out to be simple, we can give it an analogous nomological interpretation. Although it is a function on configuration space, we do not need to treat it as part of the ontology but only part of the law system. I call this a *strong nomological interpretation*, for it affords the same status to the wave function as it does to the classical Hamiltonian.

However, generic wave functions of quantum systems are very complex. But there are reasons to be optimistic. Goldstein and Zanghì (2013) have offered one. The universal wave function can be quite distinct from the wave functions of the subsystems. If we were to extend quantum mechanics to quantum gravity, then it is possible that the wave function of the universe will be stationary. This is seen in the Wheeler-DeWitt equation of canonical quantum gravity:

$$\hat{H}\Psi = 0 \tag{11}$$

[14] To borrow a term from Barry Loewer and Tim Maudlin (p.c.), on this view, the wave function is part of the *nomology* of the theory.

If we understand this equation as telling us about the time evolution of the wave function, then it tells us that the wave function does not change over time, i.e. it is stationary. Since the Schrödinger equation governs how the wave function changes over time, it is to be treated not as a fundamental equation but only as an effective equation—describing the behavior of subsystems.

It is plausible to think that a stationary wave function contain many symmetries, because usually only symmetrical wave functions are stationary. Such symmetries might ensure that the wave function is simple. For example, a function on physical space with translational invariance can only be the constant functions, which are relatively simple. Therefore, if the wave function of the universe satisfies (11), and if we understand it as telling us about time evolution, it is plausible that the universal wave function is simple. On the Bohmian theory, then, the universal wave function can be treated on a par with laws of nature.

This nomological interpretation faces some challenges. First, it is controversial whether the Wheeler-DeWitt equation governs the universal wave function. For example, there are research programs in quantum gravity that do not depend on it. Second, since the wave function is stationary, it requires some revisions about how we think about the arrow of time.[15] Chen (2018) develops a new framework of quantum mechanics that avoids these problems. However, in that approach, the fundamental quantum state has to be represented by a density matrix instead of a wave function. Allori (2017) proposes a new argument for the nomological interpretations based on symmetry principles.

These interpretations of the wave function are most compelling in the Bohmian framework. However, if we add primitive ontologies, such as mass-densities and flashes, to Everettian and GRW theories, their wave functions can also be given nomological interpretations.

4.2 Weak Nomological Interpretations

There is a growing literature on the nomological interpretation of the wave function. However, much of that is directed at a weaker thesis, which I will call the *weak nomological interpretation*. On this view, the wave function does not need to be like the classical Hamiltonian to fit into the law system. It recommends a weaker criterion on being nomological. This idea is most plausible in some extended Humean framework. In the original Humean framework, laws of nature are the axioms of the best system that summarizes the mosaic. In Loewer (2001), the Humean framework has been extended to allow for deterministic "chances." In Hall (2015), it has been further extended to allow intrinsic properties such as mass and charge to be non-fundamental and to be merely part of the best system.

According to the weak nomological interpretation (Humean version), what is

[15]The problem is that in standard Boltzmannian quantum statistical mechanics, the arrow of time is associated with the increase of entropy of the quantum system, which is a property of the wave function. Perhaps the Bohmian approach can help by providing an alternative definition of entropy in terms of particle configurations. But that has not been done.

fundamental is just the distribution of matter in the four-dimensional spacetime, and the wave function is just a dynamical variable that assists in a simple and informative summary of the mosaic. (See Miller (2014), Esfeld (2014), Bhogal and Perry (2015), Callender (2015), and Esfeld and Deckert (2017).) Although the wave function is part of the best system, it does not have to be simple *simpliciter*. It just needs to be the simplest one among all competitors. Even though the exact specification of the wave function is complicated, the best system involving the wave function might still be the simplest overall. Albert (p.c.), Maudlin (p.c.), and Dewar (2017) have raised the worry that the complete specification of particle trajectories, which will form another system, seems to postulate less information than the wave function. This is because the particle trajectories form a single curve in the configuration space, while the wave function consists in the assignment of values to every point in the configuration space. Moreover, they have raised the worry that the wave function does not supervene on the particle trajectories, since *prima facie* the particle trajectories do not determine the exact values of the wave function. However, it is true that physicists who have access only to position facts nonetheless postulate wave functions to make explanations and predictions, and they often agree on the exact wave function of the system. So the best system evaluation (and supervenience) may be more complicated than what the debate has assumed.

At any rate, the weak nomological interpretation demands less of the wave function of the universe. It does not have to be a simple function or determined in a simple way. It can be very complex, as long as it is the simplest among all the choices. The weak nomological interpretation is less realist than the previous approaches, but it could still be realist if the extended Humean model can be understood as a realist view about laws and properties.

5 The *Sui Generis* Interpretation

It is possible to be not persuaded by any of the above strategies. The high-dimensional field interpretation and the Hilbert space interpretation require sophisticated stories about the emergence of the apparent three-dimensional objects and ordinary spacetime. The low-dimensional multi-field interpretation and the subsystem property interpretation may seem to trying too hard to squeeze the wave function into familiar ontological categories.

Perhaps the lesson of quantum mechanics is that the wave function does not fit into any familiar categories of things; it is a new kind of entity. Perhaps it is neither ontological or nomological. In that case, the wave function has its own category of existence that is distinct from anything we have considered. In other words, the wave function is ontologically *sui generis*. Maudlin (2013) suggests that we should be open to that possibility.

6 Conclusion

In this article, we have surveyed three kinds of realist interpretations of the wave function: the ontological interpretations, the nomological interpretations, and the *sui generis* interpretation. A century after the discovery of quantum mechanics, although there is no consensus on what it means, we have made significant progress in constructing several realist interpretations. Almost every interpretation requires further developments, and it is too early to say which one is the best or the most fruitful interpretation. It is also too early to think that those are exhaustive of all the options available to the realist. In all likelihood, there will be other new ways to think about the wave function, from the realist perspective, that we have never considered.[16]

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[16]For an alternative perspective about the discussion of realism about the wave function, see Halvorson (2018).

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