Time's Arrow in a Quantum Universe: On the Status of Statistical Mechanical Probabilities

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Abstract

In a quantum universe with a strong arrow of time, it is standard to postulate that the initial wave function started in a particular macrostate—the special low-entropy macrostate selected by the Past Hypothesis. Moreover, there is an additional postulate about statistical mechanical probabilities according to which the initial wave function is a "typical" choice in the macrostate (the Statistical Postulate). Together, they give rise to a probabilistic version of the Second Law of Thermodynamics: typical initial wave functions will increase in entropy. Hence, there are two sources of randomness in such a universe: the quantum-mechanical probabilities in the Born rule and the statistical mechanical probabilities in the Statistical Postulate.

I propose another way to understand time's arrow in a quantum universe. It is based on what I call the Thermodynamic Theories of Quantum Mechanics. According to this perspective, there is a *natural* choice for the initial quantum state of the universe, which is given not by a wave function but by a density matrix. Moreover, the density matrix is exactly the (normalized) projection operator onto the Past Hypothesis macrostate (represented by a subspace in the energy shell). Effectively, there is a *unique* choice for the initial state. This enables a new and general strategy to eliminate statistical mechanical probabilities in the fundamental physical theories, which reduces the two sources of randomness to only the quantum mechanical one. However, the degree of uniqueness of the initial quantum state depends on the sharpness of the Past Hypothesis.

Keywords: time's arrow, Past Hypothesis, Statistical Postulate, the Mentaculus Vision, typicality, unification, foundations of probability, quantum statistical mechanics, wave function realism, quantum ontology, density matrix, initial condition of the universe

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1 Introduction

In a quantum universe with a strong arrow of time, it is standard to attribute the temporal asymmetry to a special boundary condition. This boundary condition is a macrostate of extremely low entropy. David Albert (2000) calls it the Past Hypothesis. The quantum version of the Past Hypothesis dictates that the initial wave function of the universe lies within the low-entropy macrostate. Mathematically, the macrostate is represented by a low-dimensional subspace of the total Hilbert space, which corresponds to low Boltzmann entropy.

The Past Hypothesis is accompanied by another postulate about statistical mechanical probabilities according to which the initial wave function is a "typical" choice in the macrostate (the Statistical Postulate). Together, the Past Hypothesis and the Statistical Postulate give rise to a probabilistic version of the Second Law of Thermodynamics: typical initial wave functions will increase in entropy.

The standard theory, which has its origin in Boltzmann and has been substantially developed, is a simple and elegant way of understanding time's arrow in a quantum universe. It has two characteristics:

1. Although the theory restricts the choices of initial quantum states of the universe, it does not select a unique one.

2. The theory postulates statistical mechanical probabilities (or a typicality measure) on the fundamental level. They are *prima facie* on a par with and in addition to quantum mechanical probabilities.

Hence, there are two sources of randomness in such a universe: the quantum-mechanical probabilities in the Born rule and the statistical mechanical probabilities in the Statistical Postulate. It is an interesting conceptual question how we should understand the two kinds of probabilities in the standard theory.

I propose another way to understand time's arrow in a quantum universe. It is based on what I call the Thermodynamic Theories of Quantum Mechanics (TQM). According to this perspective, there is a *natural* choice for the initial quantum state of the universe, which is given not by a wave function but by a density matrix. Moreover, the density matrix is exactly the (normalized) projection operator onto the Past Hypothesis macrostate (represented by a subspace in the energy shell). Effectively, there is a *unique* choice for the initial state. To contrast with the standard theory, TQM has the following two characteristics:

1. The theory selects a unique initial quantum state of the universe.
2. The theory does not postulate statistical mechanical probabilities (or a typicality measure) on the fundamental level.

That is, TQM enables a new and general strategy for eliminating statistical mechanical probabilities at the fundamental level. However, as I explain below, the degree of uniqueness of the initial quantum state depends on the sharpness of the Past Hypothesis.

Here is the roadmap of the paper. In §2, I will review the standard (Boltzmannian) way of understanding temporal asymmetry in classical and quantum mechanics. In §3, I introduce the perspective of the Thermodynamic Theories of Quantum Mechanics. In particular, I formulate the Initial Projection Hypothesis, which is the key component of TQM and an alternative to the Past Hypothesis. I also discuss the degrees of uniqueness of the initial state, (lack of) analogues in classical statistical mechanics, implications for the status of the quantum state, Lorentz invariance, theoretical unity, generalizations to other cosmological initial conditions, and relation to other proposals in the literature.

2 Statistical Mechanics and Time's Arrow

Statistical mechanics concerns macroscopic systems such as gas in a box. It is an important subject for understanding the arrow of time. A system of gas in a box can be described by a system of N particles, with $N > 10^{20}$. If the system is governed by classical mechanics, although it is difficult to solve the equations exactly, we can still use classical statistical mechanics (CSM) to describe its statistical behaviors, such as approach to thermal equilibrium suggested by the Second Law of Thermodynamics. Similarly, if the system is governed by quantum mechanics, we can use quantum statistical mechanics (QSM) to describe its statistical behaviors. Generally speaking, there are two different views on CSM: the individualistic view and the ensemblist view. We will first illustrate the two views with CSM, which should be more familiar and will be helpful for understanding the two views in QSM.

2.1 Classical Statistical Mechanics

Let us review the basic elements of CSM on the individualistic view.[1] For concreteness, let us consider a classical-mechanical system with N particles in a box $\Lambda = [0, L]^3 \subset \mathbb{R}^3$ and a Hamiltonian $H = H(X) = H(\boldsymbol{q_1}, ..., \boldsymbol{q_N}; \boldsymbol{p_1}, ..., \boldsymbol{p_n})$.

1. Microstate: at any time t, the microstate of the system is given by a point on a $6N$-dimensional phase space,

$$X = (\boldsymbol{q_1}, ..., \boldsymbol{q_N}; \boldsymbol{p_1}, ..., \boldsymbol{p_n}) \in \Gamma_{total} \subseteq \mathbb{R}^{6N}, \tag{1}$$

   where Γ_{total} is the total phase space of the system.

2. Dynamics: the time dependence of $X_t = (\boldsymbol{q_1}(t), ..., \boldsymbol{q_N}(t); \boldsymbol{p_1}(t), ..., \boldsymbol{p_n}(t))$ is given by the Hamiltonian equations of motion:

$$\frac{d\boldsymbol{q_i}}{dt} = \frac{\partial H}{\partial \boldsymbol{p_i}}, \frac{d\boldsymbol{p_i}}{dt} = -\frac{\partial H}{\partial \boldsymbol{q_i}}. \tag{2}$$

3. Energy shell: the physically relevant part of the total phase space is the energy shell $\Gamma \subseteq \Gamma_{total}$ defined as:

$$\Gamma = \{X \in \Gamma_{total} : E \leq H(x) \leq E + \delta E\}. \tag{3}$$

   We only consider microstates in Γ.

4. Measure: the measure μ is the standard Lebesgue measure of volume $|\cdot|$ on $\mathbb{R}^{6N}$.

5. Macrostate: with a choice of macro-variables, the energy shell Γ can be partitioned into macrostates Γ_ν:

$$\Gamma = \bigcup_\nu \Gamma_\nu. \tag{4}$$

6. Unique correspondence: every phase point X belongs to one and only one Γ_ν.

7. Thermal equilibrium: typically, there is a dominant macrostate Γ_{eq} that has almost the entire volume with respect to μ:

$$\frac{\mu(\Gamma_{eq})}{\mu(\Gamma)} \approx 1. \tag{5}$$

   A system is in thermal equilibrium if its phase point $X \in \Gamma_{eq}$.

[1] Here I follow the discussion in Goldstein and Tumulka (2011). §2.2.1 and §2.2.2 do not intend to be rigorous axiomatizations of CSM and QSM. They are only summaries of the key concepts that are important for appreciating the main ideas in the later sections.

8. Boltzmann Entropy: the Boltzmann entropy of a classical-mechanical system in microstate X is given by:

$$S_B(X) = k_B \log(\mu(\Gamma(X))), \tag{6}$$

where $\Gamma(X)$ denotes the macrostate containing X. The thermal equilibrium state thus has the maximum entropy.

9. Low-Entropy Initial Condition: when we consider the universe as a classical-mechanical system, we postulate a special low-entropy boundary condition, which David Albert calls *the Past Hypothesis*:

$$X_{t_0} \in \Gamma_{PH}\ ,\ \mu(\Gamma_{PH}) \ll \mu(\Gamma_{eq}) \approx \mu(\Gamma), \tag{7}$$

where Γ_{PH} is the Past Hypothesis macrostate with volume much smaller than that of the equilibrium macrostate. Hence, $S_B(X_{t_0})$, the Boltzmann entropy of the microstate at the boundary, is very small compared to that of thermal equilibrium.

10. A central task of CSM is to establish mathematical results that demonstrate (or suggest) that $\mu-$most microstates will approach thermal equilibrium.

Above is the individualistic view of CSM in a nutshell. In contrast, the ensemblist view differs in several ways. First, on the ensemblist view, instead of focusing on the microstate of an individual system, the focus is on the ensemble of systems that have the same statistical state ρ.[2] ρ is a distribution on the energy shell, and it also evolves according to the Hamiltonian dynamics. The crucial difference lies in the definition of thermal equilibrium. On the ensemblist view, a system is in thermal equilibrium if:

$$\rho = \rho_{mc} \text{ or } \rho = \rho_{can}, \tag{8}$$

where ρ_{mc} is the microcanonical ensemble and ρ_{can} is the canonical ensemble.[3]

2.2 Quantum Statistical Mechanics

Let us now turn to quantum statistical mechanics. For concreteness, let us consider a quantum-mechanical system with N fermions (with $N > 10^{20}$) in a box $\Lambda = [0, L]^3 \subset \mathbb{R}^3$ and a Hamiltonian $\hat{H}$. I will first present the "individualistic" view followed by the "ensemblist" view of quantum statistical mechanics (QSM). The main difference

[2] Ensemblists would further insist that it makes no sense to talk about the microstate X of a system in thermodynamic equilibrium.

[3] Instead of using the Boltzmann entropy, ensemblists use the Gibbs entropy:

$$S_G(\rho) = -k_B \int_\Gamma \rho \log(\rho) dx.$$

Since $S_G(\rho_t)$ is stationary under the Hamiltonian dynamics, it is not the right kind of object for understanding the approach to thermal equilibrium in the sense of the Second Law, as we would like to have an object that can change, and, in particular, increase with time.

with CSM is that they employ different state spaces. The classical state space is the phase space while the quantum state space is the Hilbert space. However, CSM and QSM are conceptually similar, as we can see from below.[4]

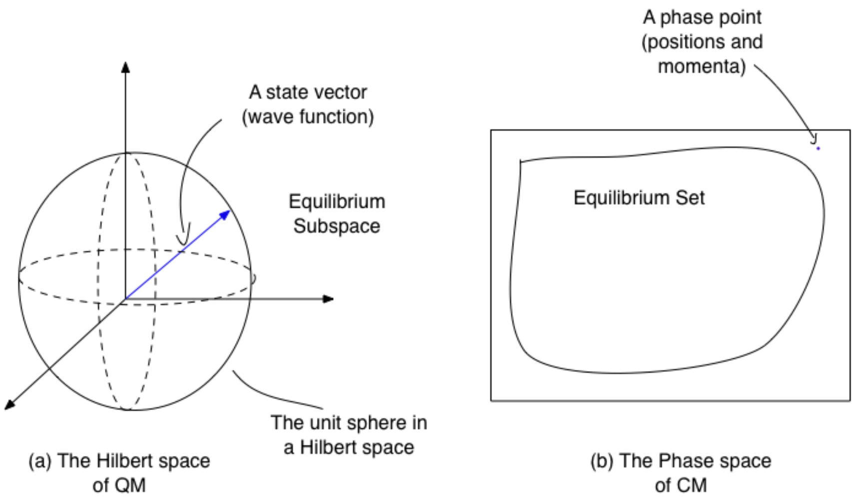


Figure 1: QM and CM employ different state spaces.

1. Microstate: at any time t, the microstate of the system is given by a normalized (and anti-symmetrized) wave function:

$$\psi(\boldsymbol{q_1}, ..., \boldsymbol{q_N}) \in \mathscr{H}_{total} = L^2(\mathbb{R}^{3N}, \mathbb{C}^k) \ , \ \| \psi \|_{L^2} = 1, \tag{9}$$

   where $\mathscr{H}_{total} = L^2(\mathbb{R}^{3N}, \mathbb{C}^k)$ is the total Hilbert space of the system.

2. Dynamics: the time dependence of $\psi(\boldsymbol{q_1}, ..., \boldsymbol{q_N}; t)$ is given by the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi. \tag{10}$$

   (In CSM, the particles move according to the Hamiltonian equations.)

3. Energy shell: the physically relevant part of the total Hilbert space is the subspace ("the energy shell"):

$$\mathscr{H} \subseteq \mathscr{H}_{total} \ , \mathscr{H} = \text{span}\{\phi_\alpha : E_\alpha \in [E, E + \delta E]\}, \tag{11}$$

   This is the subspace (of the total Hilbert space) spanned by energy eigenstates ϕ_α whose eigenvalues E_α belong to the $[E, E + \delta E]$ range. Let $D = \dim\mathscr{H}$, the number of energy levels between E and $E + \delta E$.

   We only consider wave functions ψ in $\mathscr{H}$.

[4] Here I follow the discussions in Goldstein et al. (2010a) and Goldstein and Tumulka (2011).

4. Measure: the measure μ is given by the standard Lebesgue measure on the unit sphere in the energy subspace $\mathscr{S}(\mathscr{H})$.

5. Macrostate: with a choice of macro-variables (suitably "rounded" *à la* Von Neumann (1955)), the energy shell $\mathscr{H}$ can be orthogonally decomposed into macro-spaces:
$$\mathscr{H} = \oplus_\nu \mathscr{H}_\nu \ , \ \sum_\nu \text{dim}\mathscr{H}_\nu = D \tag{12}$$
Each $\mathscr{H}_\nu$ corresponds more or less to small ranges of values of macro-variables that we have chosen in advance.

6. Non-unique correspondence: typically, a wave function is in a superposition of macrostates and is not entirely in any one of the macrospaces. However, we can make sense of situations where ψ is (in the Hilbert space norm) very close to a macrostate $\mathscr{H}_\nu$:
$$\langle\psi| P_\nu |\psi\rangle \approx 1, \tag{13}$$
where P_ν is the projection operator into $\mathscr{H}_\nu$. This means that almost all of $|\psi\rangle$ lies in $\mathscr{H}_\nu$.

7. Thermal equilibrium: typically, there is a dominant macro-space $\mathscr{H}_{eq}$ that has a dimension that is almost equal to D:
$$\frac{\text{dim}\mathscr{H}_{eq}}{\text{dim}\mathscr{H}} \approx 1. \tag{14}$$
A system with wave function ψ is in equilibrium if the wave function ψ is very close to $\mathscr{H}_{eq}$ in the sense of (13): $\langle\psi| P_{eq} |\psi\rangle \approx 1$.

   Simple Example. Consider a gas consisting of $n = 10^{23}$ atoms in a box $\Lambda \subseteq \mathbb{R}^3$. The system is governed by quantum mechanics. We orthogonally decompose the Hilbert space $\mathscr{H}$ into 51 macro-spaces: $\mathscr{H}_0 \oplus \mathscr{H}_2 \oplus \mathscr{H}_4 \oplus ... \oplus \mathscr{H}_{100}$, where $\mathscr{H}_\nu$ is the subspace corresponding to the macrostate such that the number of atoms in the left half of the box is between $(\nu - 1)\%$ and $(\nu + 1)\%$ of n. In this example, $\mathscr{H}_{50}$ has the overwhelming majority of dimensions and is thus the equilibrium macro-space. A system whose wave function is very close to $\mathscr{H}_{50}$ is in equilibrium (for this choice of macrostates).

8. Boltzmann Entropy: the Boltzmann entropy of a quantum-mechanical system with wave function ψ that is very close to a macrostate ν is given by:
$$S_B(\psi) = k_B \log(\text{dim}\mathscr{H}_\nu), \tag{15}$$
where $\mathscr{H}_\nu$ denotes the subspace containing almost all of ψ in the sense of (13). The thermal equilibrium state thus has the maximum entropy:
$$S_B(eq) = k_B \log(\text{dim}\mathscr{H}_{eq}) \approx k_B \log(D), \tag{16}$$
where eq denotes the equilibrium macrostate.

9. Low-Entropy Initial Condition: when we consider the universe as a quantum-mechanical system, we postulate a special low-entropy boundary condition on the universal wave function—the quantum-mechanical version of *the Past Hypothesis*:

$$\Psi(t_0) \in \mathscr{H}_{PH} \text{ , } \dim\mathscr{H}_{PH} \ll \dim\mathscr{H}_{eq} \approx \dim\mathscr{H} \tag{17}$$

where $\mathscr{H}_{PH}$ is the Past Hypothesis macro-space with dimension much smaller than that of the equilibrium macro-space.[5] Hence, the initial state has very low entropy in the sense of (26).

10. A central task of QSM is to establish mathematical results that demonstrate (or suggest) that μ−most (maybe even all) wave functions of small subsystems, such as gas in a box, will approach thermal equilibrium.

Above is the individualistic view of QSM in a nutshell. In contrast, the ensemblist view of QSM differs in several ways. First, on the ensemblist view, instead of focusing on the wave function of an individual system, the focus is on an ensemble of systems that have the same statistical state $\hat{W}$, a density matrix.[6] It evolves according to the von Neumann equation:

$$i\hbar\frac{d\hat{W}(t)}{dt} = [\hat{H}, \hat{W}]. \tag{18}$$

The crucial difference between the individualistic and the ensemblist views of QSM lies in the definition of thermal equilibrium. On the ensemblist view, a system is in thermal equilibrium if:

$$W = \rho_{mc} \text{ or } W = \rho_{can}, \tag{19}$$

where ρ_{mc} is the microcanonical ensemble and ρ_{can} is the canonical ensemble.[7]

For the QSM individualist, if the microstate ψ of a system is close to some macro-space $\mathscr{H}_\nu$ in the sense of (13), we can say that the macrostate of the system is $\mathscr{H}_\nu$. It is naturally associated with a density matrix:

$$\hat{W}_\nu = \frac{I_\nu}{dim\mathscr{H}_\nu}, \tag{21}$$

[5]We should assume that $\mathscr{H}_{PH}$ is finite-dimensional, in which case we can use the Lebesgue measure on the unit sphere as the typicality measure for # 10. It remains an open question in QSM about how to formulate the low-entropy initial condition when the initial macro-space is infinite-dimensional.

[6]Ensemblists would further insist that it makes no sense to talk about the thermodynamic state of an individual system.

[7]The microcanonical ensemble is the projection operator onto the energy shell $\mathscr{H}$ normalized by its dimension. The canonical ensemble is:

$$\rho_{can} = \frac{\exp(-\beta\hat{H})}{Z}, \tag{20}$$

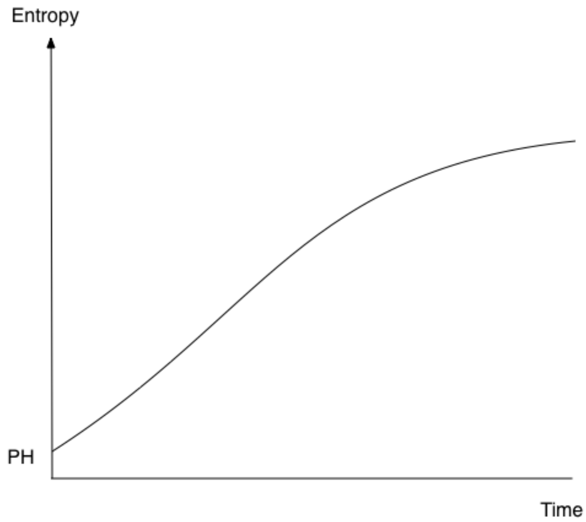
where $Z = \text{tr} \exp(-\beta\hat{H})$, and β is the inverse temperature of the quantum system.

where I_ν is the projection operator onto $\mathscr{H}_\nu$. $\hat{W}_\nu$ is also a representation of the macrostate. It can be decomposed into wave functions, but the decomposition is not unique. Different measures can give rise to the same density matrix. One such choice is $\mu(d\psi)$, the uniform distribution over wave functions:

$$\hat{W}_\nu = \int_{\mathscr{S}(\mathscr{H}_\nu)} \mu(d\psi) \left|\psi\right\rangle \left\langle\psi\right| . \tag{22}$$

In (22), $\hat{W}_\nu$ is defined with a choice of measure on wave functions in $\mathscr{H}_\nu$. However, we should not be misled into thinking that the density matrix is derivative of wave functions. What is intrinsic to a density matrix is its geometrical meaning in the Hilbert space. In the case of $\hat{W}_\nu$, as shown in the canonical description (21), it is just a normalized projection operator.

2.3 Ψ_{PH}-Quantum Theories



If we treat the universe as a quantum system, then we can distill a picture of the fundamental physical reality from the standard Boltzmannian QSM (individualistic perspective). The universe is described by a quantum state represented by a universal wave function. It starts in a Past Hypothesis macrostate (§2.2 #9) and is selected randomly according to the Statistical Postulate (§2.2 #4). It evolves by the quantum dynamics—the Schrödinger equation (§2.2 #2). That is, it has three fundamental postulates (fundamental lawlike statements[8]):

[8]Barry Loewer calls the joint system—the package of laws that includes PH and SP in addition to the dynamical laws of physics—the Mentaculus Vision. For developments and defenses of the nomological account of the Past Hypothesis and the Statistical Postulate, see Albert (2000), Loewer (2007), Wallace (2011, 2012) and Loewer (2016). Albert and Loewer are writing mainly in the context of CSM. The Mentaculus Vision is supposed to provide a "probability map of the world." As such, it requires one to take the probability distribution very seriously.

1. The Schrödinger equation.
2. The Past Hypothesis.
3. The Statistical Postulate.

However, this theory by itself faces the measurement problem. To solve the measurement problem, we can combine it with three well-known strategies:

- Bohmian mechanics (BM): the fundamental ontology in addition to the quantum state also includes point particles with precise locations, and the fundamental dynamics also includes a guidance equation that relates the wave function to the velocity of the particles.
- Everettian mechanics (S0): the fundamental ontology consists in just the quantum state evolving unitarily according to the Schrödinger equation.
- GRW spontaneous collapse theory (GRW0): the fundamental ontology consists in just the quantum state evolving by the Schrödinger equation, but the unitary evolution is interrupted by a spontaneous collapse mechanism.

The universal wave function Ψ is central to standard formulations of the above theories. The Past Hypothesis and the Statistical Postulate need to be added to them to account for the arrow of time. Let us label them Ψ_{PH}-BM, Ψ_{PH}-S0, and Ψ_{PH}-GRW0. They are all instances of what I call Ψ_{PH}-quantum theories.

There are also Everettian and GRW theories with fundamental ontologies in physical space-time, such as Everettian theory with a mass-density ontology (S0), GRW theory with a mass-density ontology (GRWm), and GRW theory with a flash ontology (GRWf). See Allori et al. (2008, 2010) for discussions. Thus, we can also have Ψ_{PH}-Sm, Ψ_{PH}-GRWm, and Ψ_{PH}-GRWf.

3 The Thermodynamic Theories of Quantum Mechanics

The Ψ_{PH}-quantum theories attempt to solve the measurement problem and account for the arrow of time. However, each Ψ_{PH}-quantum theory admits many choices for the initial quantum states. This is because the Past Hypothesis subspace $\mathscr{H}_{PH}$, although being small compared to the full energy shell, is still compatible with many wave functions.

Can we do better? Can we narrow down the choices to exactly one? I believe that we can. It is made possible in two steps:

1. Allowing the universe to be in a mixed state represented by a density matrix.
2. Choosing a natural density matrix associated with the Past Hypothesis subspace $\mathscr{H}_{PH}$.

The natural density matrix will define a new class of quantum theories, which we will call the *Thermodynamic Theories of Quantum Mechanics*. As we explain below, all of them will effectively have a unique initial quantum state.

3.1 The Initial Projection Hypothesis

The standard approach to the foundations of quantum mechanics assumes that the universe is described by a *pure* quantum state $\Psi(t)$, which is represented as a vector in the (energy shell of the) Hilbert space of the system. However, it is also possible to describe the universe by a *mixed* quantum state $W(t)$, which has its own geometric meaning in the Hilbert space.

On the latter perspective, the quantum state of the universe is in a mixed state W, which becomes the central object of the quantum theory. W obeys its own unitary dynamics—the von Neumann equation (18), which is a generalization of the Schrödinger equation (10). Moreover, we can write down density-matrix versions of Bohmian mechanics, Everettian mechanics, and GRW spontaneous collapse theory. (See Appendix for the mathematical details.) I call this view *Density Matrix Realism*.[9].

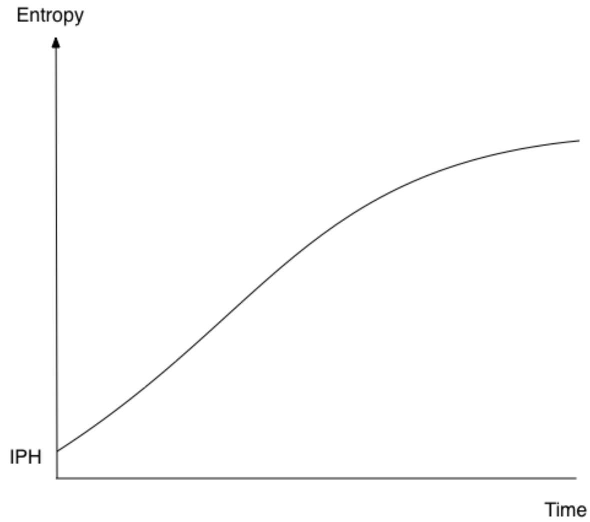
According Density Matrix Realism, the initial quantum state is also represented by a density matrix. Now, we know that to account for the arrow of time we need to postulate a low-entropy initial condition—the Past Hypothesis. Moreover, under Wave Function Realism, it is also necessary to postulate a Statistical Postulate. However, if the initial quantum state is a density matrix, then there is a natural choice for it—the normalized projection operator onto $\mathscr{H}_{PH}$:

$$\hat{W}_{PH}(t_0) = \frac{I_{PH}}{dim \mathscr{H}_{PH}}, \tag{23}$$

where t_0 represents a temporal boundary of the universe, I_{PH} is the projection operator onto $\mathscr{H}_{PH}$, dim counts the dimension of the Hilbert space, and $dim \mathscr{H}_{PH} \ll dim \mathscr{H}_{eq}$. Since the quantum state at t_0 has the lowest entropy, we call t_0 the initial time. We shall call (23) the *Initial Projection Hypothesis* (IPH). In words: the initial density matrix of the universe is the normalized projection onto the PH-subspace.

The projection operator onto $\mathscr{H}_{PH}$ contains no more and no less information than what is contained in the subspace itself. There is a natural correspondence between a subspace and its projection operator. If we specify the subspace, we know what its projection operator is, and vice versa. Since the projection operator onto a subspace carries no more information than that subspace itself, the projection operator is no more complex than $\mathscr{H}_{PH}$. This is different from Ψ_{PH}, which is confined by PH to be a vector inside $\mathscr{H}_{PH}$. A vector carries more information than the subspace it belongs to, as specifying a subspace is not sufficient to determine a vector. For example, to determine a vector in an 18-dimensional subspace of a 36-dimensional vector space, we need 18 coordinates in addition to specifying the subspace. The higher the dimension of the subspace, the more information is needed to specify

[9]This idea may be unfamiliar to some people, as we are used to take the mixed states to represent our *epistemic uncertainties of the actual pure state* (a wave function). The proposal here is that the density matrix directly represents the actual quantum state of the universe; there is no further fact about which is the actual wave function. In this sense, the density matrix is "fundamental." In fact, this idea has come up in the foundations of physics. See, for example, Dürr et al. (2005), Maroney (2005), Wallace (2011, 2012), and Wallace (2016). I discuss and defend this idea in more detail in Chen (2018a)



the vector. If PH had fixed Ψ_{PH} (the QSM microstate), it would have required much more information and become a much more complex posit. PH as it is determines Ψ_{PH} only up to an equivalence class (the QSM macrostate).

Thus, I propose that we replace the Past Hypothesis with the Initial Projection Hypothesis, the Schrödinger equation with the von Neumann equation, and the universal wave function with a universal density matrix. In contrast to Ψ_{PH} theories, there are only two corresponding fundamental postulates in W_{PH} theories:

1. The von Neumann equation.

2. The Initial Projection Hypothesis.

Notice that IPH defines a unique initial quantum state. The quantum state $\hat{W}_{PH}(t_0)$ is informationally equivalent to the constraint that PH imposes on the initial microstates. Assuming that PH selects a unique low-entropy macrostate, $\hat{W}_{PH}(t_0)$ is singled out by the data in PH (we will come back to this point in §3.2). Consequently, on the universal scale, we do not need to impose an additional probability or typicality measure on the Hilbert space. $\hat{W}_{PH}(t_0)$ is mathematically equivalent to an integral over projection onto each normalized state vectors (wave functions) compatible with PH *with respect to a Lebesgue measure*. Of course, we are not defining $\hat{W}_{PH}(t_0)$ in terms of state vectors. Rather, we are thinking of $\hat{W}_{PH}(t_0)$ as a geometric object in the Hilbert space: the (normalized) projection operator onto $\mathscr{H}_{PH}$. That is the *intrinsic* understanding of the density matrix.

As before, the above theory by itself faces the measurement problem. To solve the measurement problem, we can combine it with the well-known strategies of BM, EQM, and GRW. We will label them as W_{PH}-BM, W_{PH}-EQM, and W_{PH}-GRW. (See the Appendix for mathematical details.) Together, we will call them W_{PH}-quantum theories. Moreover, since they are motivated by considerations of the thermodynamic macrostate of the early universe, we also call them the *Thermodynamic Theories of Quantum Mechanics* (TQM).

In W_{PH}-quantum theories, the density matrix takes on a central role as the quantum microstate. Besides the low-entropy initial condition, it is also necessary to reformulate some definitions in quantum statistical mechanics:

6′ Non-unique correspondence: typically, a density matrix is in a superposition of macrostates and is not entirely in any one of the macrospaces. However, we can make sense of situations where W is very close to a macrostate $\mathscr{H}_\nu$:

$$\mathrm{tr}(WP_\nu) \approx 1, \tag{24}$$

where P_ν is the projection operator onto $\mathscr{H}_\nu$. This means that almost all of W is in $\mathscr{H}_\nu$.

7′ Thermal equilibrium: typically, there is a dominant macro-space $\mathscr{H}_{eq}$ that has a dimension that is almost equal to D:

$$\frac{\dim \mathscr{H}_{eq}}{\dim \mathscr{H}} \approx 1. \tag{25}$$

A system with density matrix W is in equilibrium if W is very close to $\mathscr{H}_{eq}$ in the sense of (24): $\mathrm{tr}(WP_{eq}) \approx 1$.

8′ Boltzmann Entropy: the Boltzmann entropy of a quantum-mechanical system with density matrix W that is very close to a macrostate ν is given by:

$$S_B(W) = k_B \log(\dim \mathscr{H}_\nu), \tag{26}$$

where $\mathscr{H}_\nu$ denotes the subspace containing almost all of W in the sense of (24).

For the Bohmian version, we have the additional resource of the particle configuration, which can enable us to further define the effective macrostate and the effective Boltzmann entropy of a quantum system in a way that is analogous to the effective wave function of the system.[10]

3.2 Uniqueness and Sharpness

Unlike the Past Hypothesis, the Initial Projection Hypothesis selects a unique quantum state. Hence, there is no need to introduce a probability or typicality measure over initial quantum states in order to neglect the anti-entropic ones. However, the uniqueness of the choice is dependent on the Past Hypothesis. That is, given the subspace $\mathscr{H}_{PH}$, there is a a unique (normalized) projection operator.

[10] Here is a flat-footed way of doing it. First, we start from the wave-function picture of BM. If Ψ_i is the effective wave function of the universe, and if Ψ_i is almost entirely in $\mathscr{H}_\nu$, then the universe is in an effective macrostate of $\mathscr{H}_\nu$ and it has effective Boltzmann entropy of $k_B \log(\dim \mathscr{H}_\nu)$. Second, we note that we can define an analogous notion of the effective density matrix of the universe. Third, we can transpose the first point to the density-matrix perspective. If W_i is the effective density matrix of the universe, and if W_i is almost entirely in $\mathscr{H}_\nu$, then the universe is in an effective macrostate of $\mathscr{H}_\nu$ and it has effective Boltzmann entropy of $k_B \log(\dim \mathscr{H}_\nu)$. However, further analysis is required.

Hence, the degree to which this choice is unique depends on the degree to which the Past Hypothesis selects a unique subspace $\mathscr{H}_{PH}$. In this section, I discuss three versions of Past Hypothesis: the Strong PH, the Weak PH, and the Fuzzy PH. On the first two, there is a unique subspace and hence a unique initial quantum state. On the last one, there is a unique subspace given an admissible precisification. I also provide some reasons why it might be an open question which version is the best one.

The Strong Past Hypothesis relies on a particular decomposition of the energy shell into macrostates (§2.2 #5). It selects a unique macrostate—$\mathscr{H}_{PH}$. The initial wave function has to start from $\mathscr{H}_{PH}$.

Strong Past Hypothesis

$$\Psi(t_0) \in \mathscr{H}_{PH} \tag{27}$$

Moreover, it is a well-defined probability space on which we can impose the Lebesgue measure (on the unit sphere of vectors). If the Strong PH makes into the Ψ_{PH} theories, then we can impose a similarly strong IPH in the W_{PH} theories:

Strong Initial Projection Hypothesis

$$\hat{W}_{PH}(t_0) = \frac{I_{PH}}{dim \mathscr{H}_{PH}} \tag{28}$$

The Strong IPH selects a unique initial quantum state, which does not require a further specification of statistical mechanical probabilities.

However, there may be several low-dimensional subspaces that can do the job of low-entropy initial condition equally well. They do not have to be orthogonal to each other. They may arise from slightly different decompositions of the energy shell into macrostates. One reason is that the Past Hypothesis is formulated in macroscopic language which is not maximally precise. The inherent imprecision can result in non-uniqueness of the initial subspace. There are two kinds of non-uniqueness: (1) there is a set of subspaces to choose from, and (2) the boundary between what is admissible and what is not is fuzzy; some subspaces are admissible but some others are borderline cases.

The first kind corresponds to the Weak Past Hypothesis:

Weak Past Hypothesis

$$\Psi(t_0) \in \mathscr{H}_1 \text{ or } \Psi(t_0) \in \mathscr{H}_2 \text{ or ... or } \Psi(t_0) \in \mathscr{H}_k \tag{29}$$

For the Weak PH, there is the corresponding Weak IPH:

Weak Initial Projection Hypothesis

$$\hat{W}_{WPH}(t_0) = \frac{I_{WPH}}{dim \mathscr{H}_{WPH}} \tag{30}$$

where $\mathscr{H}_{WPH}$ is obtained by summing $\mathscr{H}_0$ to $\mathscr{H}_k$ and taking the closure. The intuitive idea is to first take the admissible initial subspaces and then form a bigger subspace from them. This results in a determinate initial subspace, which corresponds to a natural quantum state—its normalized projection operator. Hence, the Weak IPH does not require the further specification of statistical mechanical probabilities.

The second kind corresponds to the Fuzzy Past Hypothesis:

Fuzzy Past Hypothesis

The universal wave function started in some low-entropy state. The boundary of the macrostate is vague. (The Fuzzy PH does not determinately pick out any set of subspaces.) Given the Fuzzy PH, we can formulate a Fuzzy IPH:

Fuzzy Initial Projection Hypothesis

$$\hat{W}_{FPH}(t_0) = \frac{I_{\mathscr{H}_i}}{dim \mathscr{H}_i} \tag{31}$$

where $\mathscr{H}_i$ is an admissible precisification of the Fuzzy PH. This does not select a unique initial quantum state, since there are many admissible ones. However, the Fuzzy IPH still does not require the further specification of statistical mechanical probabilities. To see why, consider the relation between the Fuzzy PH and the Statistical Postulate. The latter requires a probability space to define the measure. The measure can only be defined relative to an admissible precisification. That is, the measure is over possible state vectors in a precise state space. In other words, the statistical mechanical probabilities are only meaningful relative to a precisification of the initial subspace. To be sure, the exact probability may be empirically inaccessible to us — what we have access to may only be certain interval-valued probabilities.[11] However, without the precisification of the boundary, we do not even have an interval-valued probabilities, as where to draw the boundary is unsettled. Given this fact, we can see that the original statistical mechanical probabilities require admissible precisifications—the statement will have to be:

- For a choice of an initial subspace, and given the Lebesgue measure μ on it, μ−most (maybe even all) wave functions in that subspace will approach thermal equilibrium.

So the corresponding statement for the Fuzzy IPH will be:

- Every admissible density matrix will approach thermal equilibrium, where an admissible density matrix is a normalized projection operator onto an admissible initial subspace.

Since every admissible density matrix will approach thermal equilibrium, there is no need for an additional probability measure over initial quantum states in order to neglect the anti-entropic states.

To summarize: all three versions of the IPH get rid of the fundamental statistical mechanical probabilities. Both the Strong and the Weak IPH select a unique initial

[11]See Bradley (2016) for a review of imprecise probabilities.

quantum state. Although the Fuzzy IPH does not, the initial state is unique given an admissible precisification of the PH subspace.

Are there any reasons for preferring one version of IPH to the other two? I believe that is an open question. It depends on which one figures into the best systematization of the empirical facts. Presumably, the best system summarizing the distribution of matter will need to balance simplicity and strength (informativeness). This is independent of considerations of Humeanism vs. Anti-Humeanism. For Humeans, being in the best system constitutes being a law of nature. For Anti-Humeans, being in the best system is evidence for being a law of nature. The more precise the initial condition is, the more informative it becomes. However, the precision required to state the initial condition may incur cost of complexity, since the macroscopic language (which is simple) will need to be sharpened by more microscopic language (which can be complex to state explicitly). In this sense, there is a trade-off between simplicity and informativeness. Hence, it is not clear which version will win the competition.

However, there are additional reasons to not like the fuzzy version. Fundamental laws of physics and fundamental postulates in the theory are usually taken to be precise and non-vague. Vagueness is often symptom that something has gone wrong—such as the notion of "measurement" and "observation" in textbook versions of quantum mechanics (Bell (1990)). Therefore, other things being equal, we should prefer a theory that is formulated with less vague notions. The Strong IPH and the Weak IPH are to be preferred, other things being equal, to the Fuzzy IPH. But this is by no means a decisive argument. In future work, I will discuss this point in more detail.

The Strong IPH and the Weak IPH have another interesting consequence. It will make the Everettian theory strongly deterministic in the sense of Penrose (1999): Strong determinism "is not just a matter of the future being determined by the past; *the entire history of the universe is fixed*, according to some precise mathematical scheme, *for all time.*" This is because the Everettian theory is deterministic for all beables (local or non-local). With the help of a unique initial state, there is no ambiguity at all about what the history could be—there is only one way for it to go: starting from $\hat{W}_{PH}(t_0)$ (on Strong IPH) or $\hat{W}_{FPH}(t_0)$ (on Weak IPH) and evolving by the von Neumann equation.

3.3 Other Applications

Getting rid of statistical mechanical probabilities at the fundamental level is not the only advantage of W_{PH}-quantum theories over Ψ_{PH}-quantum theories. I discuss them in more detail in Chen (2018a,b). Here I briefly summarize the other features of W_{PH}-quantum theories:

1. The meaning of the quantum state.

   It has been a long-standing puzzle regarding how to understand the meaning of the quantum state, especially because it is defined as a function on a high-dimensional space and it is a non-separable object in physical space. A particularly attractive proposal is to understand it as nomological—being on a par with laws of nature. However, it faces a significant problem since typical wave functions are highly complex and not simple enough to be nomological. In contrast, if the quantum state is given by IPH, the initial quantum state will inherit the simplicity of the PH subspace. If PH is simple enough to be a law, then the initial quantum state is simple enough to be nomological. This makes possible the nomological conception without relying on specific proposals about quantum gravity (cf: Goldstein and Zanghì (2013)) or weakening our conception of laws of nature (cf: Miller (2014), Esfeld (2014), Bhogal and Perry (2015), Callender (2015), and Esfeld and Deckert (2017)). However, I should emphasize that the nomological interpretation is not the only way to understand the density matrix theory, as other proposals are also valid, such as the high-dimensional field and the low-dimensional multi-field interpretations.

2. Lorentz invariance.

   David Albert (2012) observes that there is a logical conflict among three things: quantum entanglement, Lorentz invariance, and what he calls narratability. A world is narratable just in case its entire history can be narrated in a linear sequence and every other sequence is merely a geometrical transformation from that. Since narratability is highly plausible, denying it carries significant cost. Hence, the real conflict is between quantum entanglement (a purely kinematic notion) and Lorentz invariance. This applies to all quantum theories that take quantum entanglement to be fundamental. However, given IPH, it is possible to take the nomological interpretation of the quantum state and remove entanglement facts among the facts about the distribution of physical matter. This is especially natural for W_{PH}-Sm, somewhat less naturally to W_{PH}-GRWm and W_{PH}-GRWf, and potentially applicable to a fully Lorentz-invariant version of W_{PH}-BM.

3. Kinematic and dynamic unity.

   In Ψ_{PH}-quantum theories, many subsystems will not have pure-state descriptions by wave functions due to the prevalence of entanglement. Most subsystems can be described only by a mixed-state density matrix, even when the universe as a whole is described by a wave function. In contrast, in W_{PH}-quantum theories, there is more uniformity across the subsystem level and the universal level: the universe as a whole as well as most subsystems are described by the same kind of object—a (mixed-state) density matrix. Since state descriptions concern the kinematics of a theory, we say that W_{PH}-quantum theories have more *kinematic unity* than their Ψ-counterparts:

   KINEMATIC UNITY The state description of the universe is of the same kind as the state descriptions of most quasi-isolated subsystems.

   Moreover, in a universe described by Ψ_{PH}-BM, subsystems sometimes do not have conditional wave functions due to the presence of spin. In contrast, in a universe described by W_{PH}-BM, the universe and the subsystems always have quantum states given by density matrices. This is because we can always define conditional density matrix for a Bohmian subsystem. (See Appendix for detail.) That is, in W_{PH}-BM, the W-guidance equation is always valid for the universe and the subsystems. In Ψ_{PH}-BM, the wave-function version of the guidance equation is not always valid. Thus, the W-BM equations are valid in more circumstances than the BM equations. We say that W_{PH}-BM has more dynamic unity than Ψ_{PH}-BM:

Dynamic Unity The dynamical laws of the universe are the same as the effective laws of most quasi-isolated subsystems.

Both kinematic unity and dynamic unity come in degrees, and they are only defeasible reasons to favor one theory over another. But it is nonetheless interesting that merely adopting the density-matrix framework will make the theory more "unified" in the above senses.

3.4 Generalizations

IPH is not the only principle that leads to the selection of a unique or effectively unique initial quantum state of the universe. It is just one example of a simple principle. It is easy to generalize the strategy discussed here to other simple principles about the cosmological initial condition:

- Start from the full Hilbert space (energy shell) $\mathscr{H}$.
- Use simple principles (if there are any) to determine an initial subspace $\mathscr{H}_0 \subset \mathscr{H}$.
- Choose the natural quantum state in that subspace—the normalized projection $\hat{W}_0(t_0) = \frac{I_0}{dim \mathscr{H}_0}$.
- The natural choice will be simple and unique.

Another related but different example is the quantum version of the Weyl Curvature Hypothesis proposed by Ashtekar and Gupt (2016) based on the proposal of the classical version in Penrose (1999). However, Ashtekar and Gupt's hypothesis results in an infinite-dimensional unit ball of initial wave functions, which may not be normalizable. It is not clear to me whether there will be significant dimension reduction if we intersect it with the energy shell. In any case, the problem of non-normalizability is a general problem in cosmology which may require an independent solution.

3.5 Other Proposals

I would like to contrast my proposal with three other proposals in the literature.

Albert (2000) proposes that it is plausible that Ψ_{PH}-GRW theories does not need the Statistical Postulate. This is because the GRW jumps may be large enough (in Hilbert space) to render every initial wave function entropic in a short time. An anti-entropic wave function of a macroscopic system that evolves forward will be quickly hit by a GRW jump. As long as the GRW jump has a certain width that is large compared to the width of the anti-entropic set, the wave function will collapse into an entropic one. This relies on a conjecture about GRW theory: the final and empirically adequate GRW models will have a collapse width that is large enough. If that conjecture can be established, then, for every initial wave function, it is with a GRW probability that it will be entropic. This may correspond to the right form of the probabilistic version of the Second Law of Thermodynamics. I believe this is a very plausible conjecture. But it is an additional conjecture nonetheless, and it only works for collapse theories. It is an empirically open question whether GRW will survive experimental tests in the next century. In contrast, my proposal is fully general—it works for GRW theories, Bohmian theories, and Everettian theories, and it does not rely on additional conjectures beyond those already postulated in QSM.

Wallace (2011) proposes that we can replace the Past Hypothesis and the Statistical Postulate with a Simple Dynamical Conjecture. In essence, it says that every Simple wave function will evolve to higher entropy. Here, "Simple" is a technical notion here meaning that the wave function is simple to specify and not specified by using time-reverse of an anti-entropic wave function. The idea is to replace the statistical postulate, which gives us reasons to neglect certain initial wave functions, with another postulate about simplicity, which also gives us reasons to set aside certain initial wave functions. This is a very interesting conjecture, which I think one should seriously investigate. But it is an additional conjecture nonetheless, although it has applicability to all quantum theories.

Wallace (2016) proposes that we can allow quantum states to be either pure or mixed. Moreover, he suggests that we can reinterpret probability distributions over wave functions as part of the state description and not an additional postulate about probability. There is much in common between Wallace's (2016) proposal and my proposal. However, the way that I get rid of statistical mechanical probabilities is not by way of a reinterpretation strategy but by using the uniqueness (or effective uniqueness) of the initial quantum state. To be sure, there are many ways to achieve the goal, and the two approaches are quite related.

I would like to briefly mention the possibility of implementing my proposal or Wallace's (2016) proposal in CSM. The upshot is that it is much less natural to do so in CSM than in QSM, for several reasons. On the classical mechanical phase space, the object that plays a similar role to the density matrix is the probability function $\rho(x)$. However, it is not clear how to understand its meaning as something ontic. If it is to be understood as a high-dimensional field and the only object in the ontology, then we have a Many-Worlds theory, unless we revise the Hamiltonian dynamics and make it stochastic. If it is to be understood as a low-dimensional multi-field (cf: Chen (2017b) and Hubert and Romano (2018)), it is not clear what corresponds to the momentum degrees of freedom. Second, the dynamics is not as natural in

CSM. If it is to be understood as a high-dimensional field guiding a point particle (the phase point), it is not clear why we increase the complexity of the ontology. If it is to be understood as a nomological object, it is not clear what role it plays in the dynamics—it certainly does not give rise to a velocity field, because that is the job of the Hamiltonian function.[12]

4 Conclusion

The Thermodynamic Theories of Quantum Mechanics (TQM or W_{PH}-quantum theories) provide a new strategy to eliminate statistical mechanical probabilities from the fundamental postulates of the physical theory. It does so in a simple way that does not rely on further conjectures about quantum mechanics or statistical mechanics. Moreover, it leads to several other applications and generalizations that we only discussed briefly above. I think the most interesting feature is that it brings together the foundations of quantum mechanics and the foundations of statistical mechanics.

Appendix

(1) Ψ_{PH}-Quantum Theories

Ψ_{PH}-Bohmian mechanics: (Q, Ψ_{PH})

The Past Hypothesis:

$$\Psi_{PH}(t_0) \in \mathscr{H}_{PH} \tag{32}$$

The Initial Particle Distribution:

$$\rho_{t_0}(q) = |\psi(q, t_0)|^2 \tag{33}$$

The Schrödinger Equation:

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \tag{34}$$

The Guidance Equation (Dürr et al. 1992):

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_i\psi(q)}{\psi(q)}(q = Q) \tag{35}$$

[12]See McCoy (2018) for a recent attempt in this direction. I think these considerations would also apply to his account, although his focus is not so much on the metaphysics but on the interpretation of CSM and QSM.

Ψ_{PH}-Everettian mechanics

The Past Hypothesis:

$$\Psi_{PH}(t_0) \in \mathscr{H}_{PH} \tag{36}$$

The Schrödinger Equation:

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \tag{37}$$

The Mass Density Equation:

$$m(x,t) = \langle\psi(t)|\,M(x)\,|\psi(t)\rangle\,, \tag{38}$$

W_{PH}-S0: only Ψ_{PH}.
W_{PH}-Sm: $m(x,t)$ and Ψ_{PH}.

Ψ_{PH}-GRW theory

The Past Hypothesis:

$$\Psi_{PH}(t_0) \in \mathscr{H}_{PH} \tag{39}$$

The linear evolution of the wave function is interrupted randomly (with rate $N\lambda$, where λ is of order 10^{-15} s^{-1}) by collapses:

$$\Psi_{T^+} = \frac{\Lambda_{I_k}(X)^{1/2}\Psi_{T^-}}{||\Lambda_{I_k}(X)^{1/2}\Psi_{T^-}||}, \tag{40}$$

with the collapse center X being chosen randomly with probability distribution $\rho(x) = ||\Lambda_{i_k}(x)^{1/2}\Psi_{T^-}||^2 dx$, where the collapse rate operator is defined as:

$$\Lambda_{I_k}(x) = \frac{1}{(2\pi\sigma^2)^{3/2}}e^{-\frac{(Q_k-x)^2}{2\sigma^2}} \tag{41}$$

where Q_k is the position operator of "particle" k, and σ is a new constant of nature of order 10^{-7} m postulated in current GRW theories.

Ψ_{PH}-GRWm and Ψ_{PH}-GRWf: defined with local beables $m(x,t)$ and F.

(2) W_{PH}-Quantum Theories

W_{PH}-Bohmian mechanics: (Q, W_{PH})

The Initial Projection Hypothesis:

$$\hat{W}_{PH}(t_0) = \frac{I_{PH}}{dim\mathscr{H}_{PH}} \tag{42}$$

The Initial Particle Distribution:

$$P(Q(t_0) \in dq) = W_{PH}(q, q, t_0)dq \tag{43}$$

The Von Neumann Equation:

$$i\hbar \frac{\partial \hat{W}}{\partial t} = [\hat{H}, \hat{W}] \tag{44}$$

The W_{PH}-Guidance Equation (Dürr et al. 2005):

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\nabla_{q_i} W_{PH}(q, q', t)}{W_{PH}(q, q', t)} (q = q' = Q) \tag{45}$$

W_{PH}-Everettian mechanics

The Initial Projection Hypothesis:

$$\hat{W}_{PH}(t_0) = \frac{I_{PH}}{dim \mathscr{H}_{PH}} \tag{46}$$

The Von Neumann Equation:

$$i\hbar \frac{\partial \hat{W}}{\partial t} = [\hat{H}, \hat{W}] \tag{47}$$

The Mass Density Equation:

$$m(x, t) = \mathrm{tr}(M(x)W(t)), \tag{48}$$

W_{PH}-S0: only W_{PH}.
W_{PH}-Sm: $m(x, t)$ and W_{PH}.

W_{PH}-GRW theory

The Initial Projection Hypothesis:

$$\hat{W}_{PH}(t_0) = \frac{I_{PH}}{dim \mathscr{H}_{PH}} \tag{49}$$

The linear evolution of the density matrix is interrupted randomly (with rate $N\lambda$, where λ is of order 10^{-15} s^{-1}) by collapses:

$$W_{T^+} = \frac{\Lambda_{I_k}(X)^{1/2} W_{T^-} \Lambda_{I_k}(X)^{1/2}}{\mathrm{tr}(W_{T^-} \Lambda_{I_k}(X))} \tag{50}$$

with X distributed by the following probability density:

$$\rho(x) = \mathrm{tr}(W_{T^-}\Lambda_{I_k}(x)) \quad (51)$$

where the collapse rate operator is defined as:

$$\Lambda_{I_k}(x) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{(Q_k - x)^2}{2\sigma^2}} \quad (52)$$

where Q_k is the position operator of "particle" k, and σ is a new constant of nature of order 10^{-7} m postulated in current GRW theories.

W_{PH}-GRWm and W_{PH}-GRWf: defined with local beables $m(x, t)$ and F.

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