Fundamental Nomic Vagueness

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Introduction

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- simple
- informative
- explanatory
- elegant
- unifying
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Are they also exact?

- Derived laws.
- Candidate fundamental laws of physics.

Introduction

- What would it take for a fundamental law to fail to be exact?
- What is nomic vagueness?
- Does nomic vagueness exist in a world like ours?
- Not all fundamental laws are mathematically expressible?
- The mathematical foundation of physics?
- The nature of laws?
- Connections to ontic vagueness, semantic vagueness, and epistemic vagueness?

Context of investigation: arrows of time and foundations of statistical mechanics.
[Implications for other domains]

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- How can any of these be true when the fundamental equations of motion (essentially) lack an arrow of time?

These may seem like **metaphysical** questions going beyond the scope of physics.

The Past Hypothesis

Following Boltzmann, Feynman, Reichenbach, and Penrose, we suggest that questions like these require a **scientific** explanation.

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A proposed explanation in the literature:

Past Hypothesis (PH) The universe 'initially' was in a low-entropy macrostate.

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A proposed explanation in the literature:

Past Hypothesis (PH) The universe 'initially' was in a low-entropy macrostate.

- There are good reasons to think that PH is an additional fundamental law of nature.
- PH is part of the scientific explanation for nomic regularities, such as the Second Law of Thermodynamics.
- PH is not derived from other laws.
- PH plays a crucial role in our reasoning about counterfactuals, records, and influence.

Talk Outline

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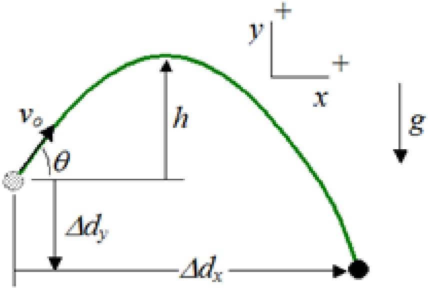

Figure: Newtonian mechanics for a projectile. Picture source: Wikipedia

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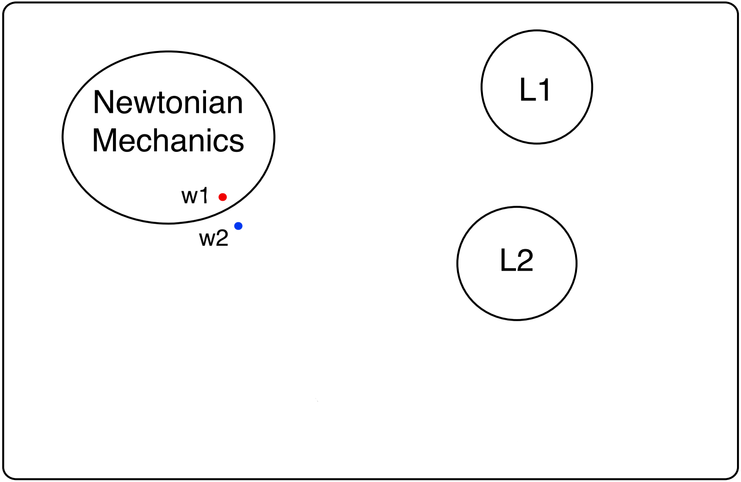
- In the space of possible worlds, there is an exact and determinate set of worlds compatible with Newtonian mechanics.
- A law L is exact if, for any world w, there is a determinate fact about whether w is compatible with L.

Exact Laws

W, the space of all possible worlds



Ideal for Exactness

- Frege's *Begriffsschrift*
- Russell's logical atomism
- Leibniz's *characteristica universalis*

Exactness and Mathematics

Mathematics:

- The ideal language; exemplar of exactness; free from vagueness.
- Set-theoretic foundation: membership relation is exact.
- Perfect language for fundamental physics, *if all the fundamental physical laws turn out to be exact*.
- What if some fundamental laws turn out to be vague?
- A tempting thought: add some degreed notion of set-membership. But it's not going to work! [Williamson (1994); Rinard (2017)]

Vague Laws

Figure: Visual imagery of a vague law. Picture source: Flickr, seiichi o

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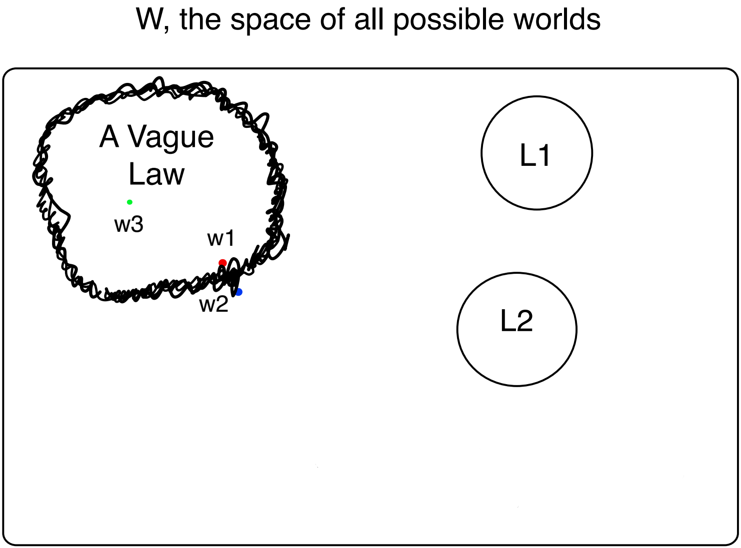
- L is a vague law if there exists some world w such that there fails to be a determinate fact about whether w is compatible with L.
- Some worlds are close to the (fuzzy) boundary of L.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A Vague Law
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A case for nomic vagueness that deserves serious consideration that may be less controversial:

- The Past Hypothesis (PH)

We will focus on PH now.
[Bonus: differences between vagueness in PH and vagueness in QM collapse axioms]

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We will try to address some of these questions in the next few slides.

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A case of Sorites paradox.

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Similarly for other vague predicates: tall, low, red, ...

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Higher-order vagueness defies mathematical modeling.

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Impossibility Conjecture It is impossible to adequately express a vague fundamental law using the language of mathematics.

Vagueness of the Past Hypothesis

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- PH is stated in the language of macrostates and macro-variables.

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Super Weak Past Hypothesis (SWPH) The universe initially was in a low-entropy state.

- How low is low?
- The harder case is the slightly stronger version that I think Albert (2012) and Loewer (2016) have in mind.

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- Perhaps one can imagine taking a union of all the admissible ones.
- But admissibility itself is also vague. [Higher-order vagueness]

The Phase Space

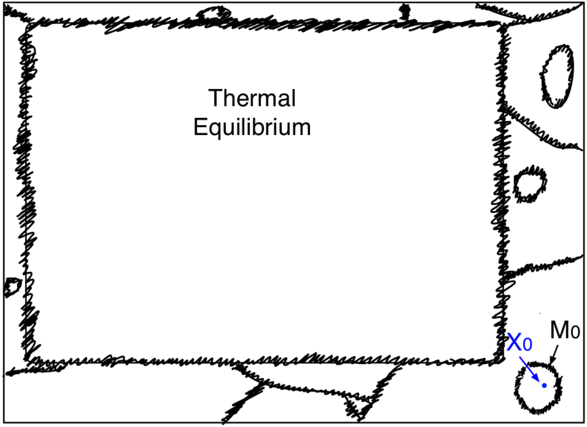

Figure: No evidence for an exact carving that strikes the best balance between simplicity and informativeness.

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Let's call them C-parameters. In practice they don't make too much of a difference as long as we are sensible in our choices.

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- A vague matter.

Cf: the Sorites paradox.

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The correspondence between macrostates and sets of microstates:

- Not exact.
- Not even "imprecise."
- It is vague.

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- Maybe nomic vagueness does not trouble you at all.

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- But in any case, we can ask whether it's possible to get rid of nomic vagueness somehow.
- Perhaps all things being equal, we might prefer a world in which the simplest and most informative description is not vague, or a world in which the fundamental laws are exact.
- Mathematical expressibility!

Getting Rid of Nomic Vagueness

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- Carroll-Chen (2004) model
- Open question

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- But there is reason to be hopeful if we are open to a new way of thinking about quantum mechanics in a time-asymmetric universe.
- And the new approach might make one more open to accept a low-entropy initial condition as a fundamental law.

Strong Past Hypothesis

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Strong Past Hypothesis (SPH) The initial microstate of the universe belongs to a precise set Γ_0.

SPH supports an epistemic interpretation of the vagueness of PH.

Strong Past Hypothesis

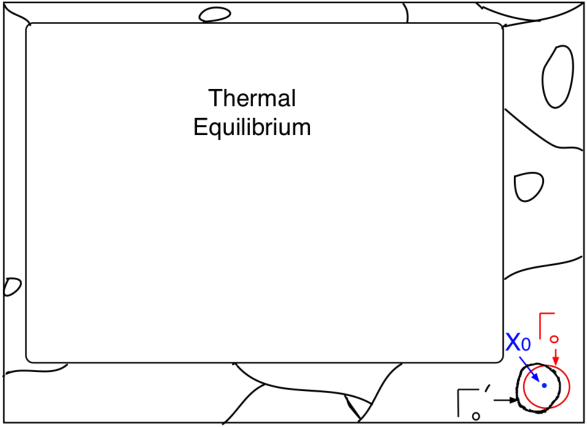

Figure: The Strong Past Hypothesis with a precise set of microstates Γ_0. The fuzziness is removed.

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- The exact choice of Γ_0 is arbitrary in an objectionable sense.
- It amounts to an exact size of cells for coarse-graining, an exact correspondence of coarse-grained distributions with thermodynamic quantities, and an exact cut-off of macrostate membership when we have quantum superposition.

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- Typically (in most worlds), any slight changes in the values of natural constants will be reflected in the material condition of the world, and they will change the nomological status of the world from possible to impossible (or some change wrt the probabilistic measure).
- Same for the exact forms of other fundamental laws.
- We call this property 'traceability.'

Traceability

Traceability-at-a-World A certain adjustable parameter O in the physical law L is traceable at world w if any change in O (while holding other parameters fixed) will result in some change in the nomological status of w with respect to L, i.e. from possible to impossible or from likely to unlikely (or some other change in the probabilistic measures).

Traceability A certain adjustable parameter O in the physical law L is traceable if O is traceable at most worlds allowed by L.

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For a typical Newtonian world whose microscopic history h is a solution to the Newtonian laws, h will not be possible given any of those changes.

Traceability

Other examples: the laws and dynamical constants of Maxwellian electrodynamics, of Bohmian mechanics, of Everettian quantum theory, of special and general relativity.

Stochastic theories: GRW.

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- Unlike natural constants, the Γ_0 in SPH is not traceable.
- Most "admissible" changes of the boundary of Γ_0 will not have any effects in typical worlds compatible with Γ_0.
- In general, you can replace an infinity of borderline worlds inside Γ_0 with another infinity of borderline worlds just outside the boundary such that there will be no differences to whether the actual world is possible or whether the actual macro-history is likely.

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- Quantum WPH: the initial wave function is in a low-entropy macrostate M_0, specified by the macro-variables S_0, V_0, T_0, P_0.
- M_0 only vaguely corresponds to sets of wave functions. It vaguely corresponds to subspaces in Hilbert space.

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Dilemma

Nomic vagueness vs. untraceable arbitrariness

- Desirable to maintain mathematical expressibility of fundamental laws
- Desirable to maintain a tight connection between nomic and ontic
- Conservativeness and continuity with history of science...not clear-cut.

The importance of traceability as a theoretical virtue: it explains why we are more ok with a vague PH than a vague theory of QM.

Principled Difference

What can be a principled reason that distinguishes the two cases?

- For WPH: its exact alternative (SPH) with precise boundaries is <u>untraceable</u>.
- For the vague measurement axiom, its exact alternative is in fact <u>traceable</u>: different cut-offs in the law will typically lead to differences in the fundamental material ontology.

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But if we can do it only if we commit untraceable arbitrariness, then a fundamental yet vague law is perfectly acceptable.

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- Density Matrix Realism: the quantum state of the universe is objective and impure.
- The QSM microstate of the universe is an impure density matrix, W.
- The density matrix W enters into the dynamical equations: it guides Bohmian particles (W-BM), or collapses spontaneously (W-GRW), or gives rise to an emergent Everettian multiverse (W-EQM).

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- We propose that there is a natural choice, given a pre-selected low-dimensional subspace $\mathscr{H}_0$.
- Initial Projection Hypothesis (IPH): the initial density matrix of the universe is the normalized projection onto $\mathscr{H}_0$.
- That is, $W_{IPH}(t_0) = \mathbb{I}_0 / \dim \mathscr{H}_0$.

Dissolving the Dilemma

This view leads to the Wentaculus package.

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For more details on its connection to topics such as quantum ontology, wave function realism, Humean supervenience, chance, and empirical equivalence, see my papers on arXiv:

- 1712.01666 (*BJPS*, 2021)
- 2006.05029 (*Noûs*, 2022)
- 1810.07010 (in Valia Allori (ed.), *Statistical Mechanics and Scientific Explanation*, World Scientific, 2020)
- 1901.08053 (2019)

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So $W_{IPH}(t_0)$, and the initial subspace $\mathscr{H}_0$, become <u>traceable</u>.

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- We can postulate IPH without committing to objectionably sharp boundaries in nature.

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Nomic vagueness disappears. Laws are also traceable.

Conclusion

- A general account of nomic exactness and nomic vagueness.
- Case study: Past Hypothesis.
- Dilemma in this case: nomic vagueness vs. untraceable arbitrariness.
- Dilemma dissolved in the Wentaculus package.
- Surprise: quantum theory actually removes vagueness.

For more details, see preprint version: arXiv 2006.05298

Conclusion

But nomic vagueness may come up elsewhere in the final theory of physics.

- Lessons from the Past Hypothesis.
- Other trade-offs; case-by-case method.
- Empiricist attitude: be open-minded; be willing to revise our old principles.
- Perhaps not all laws are exact.
- Not all laws are mathematically expressible.
- Or: new mathematical foundation for physics that refutes the impossibility conjecture.

Impossibility Conjecture It is impossible to adequately express a vague fundamental law using the language of mathematics.

Conclusion

Either way, we learn something surprising:

- Vagueness not only permeates ordinary language but can also arise in the objective nomological order;
- We need to rethink the foundations of mathematics if mathematics is the ideal language for physics.

The end.

Bonus: The Classical Case

The quantum maneuver in my approach is compatible with solutions to the measurement problem.

The classical maneuver would not be strictly parallel. It could introduce a version of the measurement problem, which requires complicating the dynamics or the ontology.

Obstacles:

- Determinism $\rightarrow$ indeterminism.
- Single world $\rightarrow$ many worlds.

Bonus: PH and SP are laws?

Four arguments for the nomological status of PH and SP.

1. Scientific explanation: laws ground laws.
2. Counterfactual asymmetry: holding certain facts fixed.
3. Reliability of records: a modal notion.
4. Humean argument: the best summary includes PH and SP.

Bonus: Empirical Equivalence

Criterion for Empirical Equivalence

Two theories A and B are empirically equivalent if at any time t, they assign the same probability distribution over all possible experimental outcomes.

- We can show this rigorously for W versions of Bohm, Everett, and GRW.
- Is it enough? Bell's jumpy Everettian world.
- Subsystem analysis.

arXiv: 1901.08053

W_{IPH}-Bohmian mechanics: (Q, W_{IPH})

The Initial Projection Hypothesis:

$$\hat{W}_{IPH}(t_0) = \frac{I_{PH}}{dim\mathscr{H}_{PH}} \tag{1}$$

The Initial Particle Distribution:

$$P(Q(t_0) \in dq) = W_{IPH}(q, q, t_0)dq \tag{2}$$

The Von Neumann Equation:

$$i\hbar\frac{\partial \hat{W}}{\partial t} = [\hat{H}, \hat{W}] \tag{3}$$

The W_{IPH}-Guidance Equation (Dürr et al. 2005):

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_{q_i} W_{IPH}(q, q', t)}{W_{IPH}(q, q', t)}(q = q' = Q) \tag{4}$$

W_{IPH}-Everettian mechanics

The Von Neumann Equation:

$$i\hbar\frac{\partial \hat{W}}{\partial t} = [\hat{H}, \hat{W}] \tag{5}$$

The Mass Density Equation:

$$m(x,t) = \mathrm{tr}(M(x)W(t)), \tag{6}$$

W_{IPH}-S0: only W_{IPH}.
W_{IPH}-Sm: $m(x,t)$ and W_{IPH}.

W_{IPH}-GRW spontaneous collapse theories

The linear evolution of the density matrix is interrupted randomly (with rate $N\lambda$) by collapses:

$$W_{T^+} = \frac{\Lambda_{I_k}(X)^{1/2} W_{T^-} \Lambda_{I_k}(X)^{1/2}}{\mathrm{tr}(W_{T^-} \Lambda_{I_k}(X))} \tag{7}$$

with X distributed by the following probability density:

$$\rho(x) = \mathrm{tr}(W_{T^-} \Lambda_{I_k}(x)) \tag{8}$$

where the collapse rate operator is defined as:

$$\Lambda_{I_k}(x) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{(Q_k - x)^2}{2\sigma^2}} \tag{9}$$

W_{IPH}-GRWm and W_{IPH}-GRWf: defined with local beables $m(x, t)$ and F.