Introduction to Major Approaches to Quantum Field Theory

Michael Miller · Philosophy and IHPST · University of Toronto

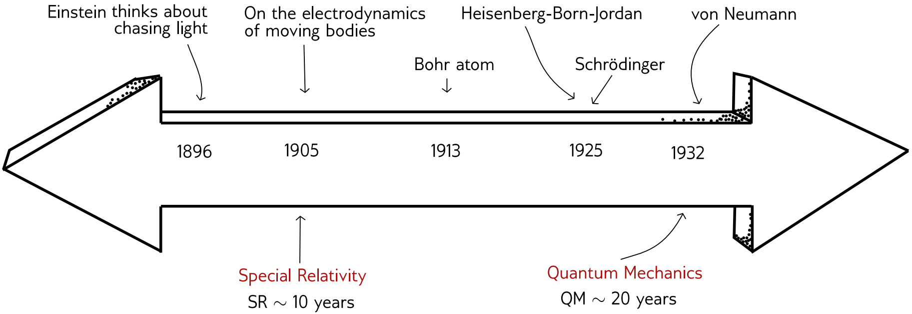
Warning: I will make no attempt here to be exhaustive!

QFT is a central pillar of modern physics and it is used to represent a wide variety of physical systems (elementary particles, condensed matter systems, early universe cosmology, quantum gravity, neuroscience, ...).

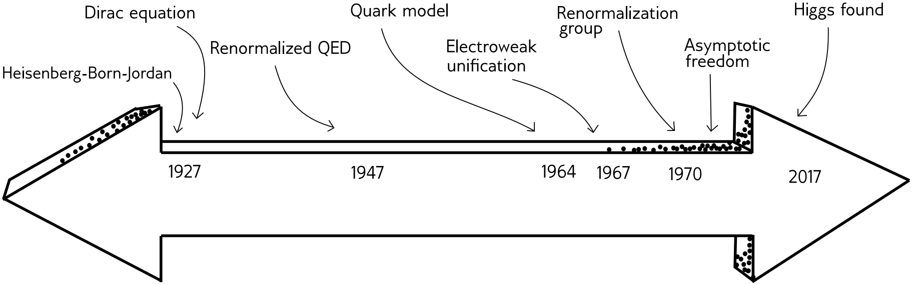
Yesterday we saw that the interpretive puzzles of quantum mechanics carry over directly to quantum field theory.

A (new?) problem: QFT strains the available accounts of what theories are, how they hold empirical content in their structure, and how they might inform or be informed by metaphysics.

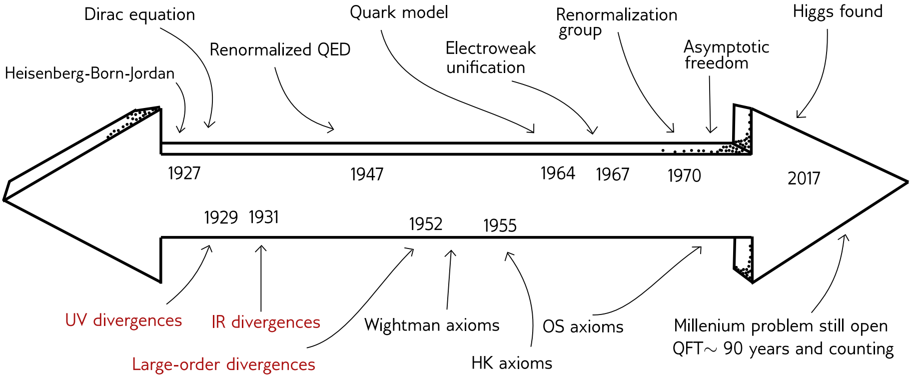
Special Relativity and Quantum Mechanics



Quantum Field Theory



Quantum Field Theory



> ...just what is a quantum field theory ...is a difficult question, since at present what we have after thirty years of intensive effort, is a collection of partially heuristic technical developments in search of a theory; but it is a natural one to examine axiomatically.

(Segal 1959, p. 341)

> Given a theory T, ... we confront the exemplary interpretive question of how exactly to establish a correspondence between T's models and worlds possible according to T. That is, we confront that question *if* T is the sort of thing that has models. 'A collection of partially heuristic technical developments' isn't readily attributed a set of models about whose underlying ontology or principles of individuation philosophical questions immediately arise. This isn't to say that 'a collection of partially heuristic technical developments' is unworthy of philosophical attention. It is in itself a philosophically provocative circumstance that such a collection can enjoy stunning empirical success.

(Ruetsche 2011, p. 102-103)

Two approaches

1) Axiomatic and constructive QFT

2) Perturbative QFT ($+$ lattice QFT, etc.)

The axiomatic approach: Write down an explicit set of mathematical conditions that any relativistic quantum field theory should satisfy.

GW axioms

- Hilbert space
- Temperedness
- Covariance
- Spectrum condition
- Micro-causality
- Uniqueness of the vacuum

W axioms

- Temperedness
- Covariance
- Positive definiteness
- Spectrum condition
- Locality
- Cluster property

OS axioms

- Temperedness
- Covariance
- Reflection positivity
- Symmetry
- Cluster property

HK axioms

- Isotony
- Covariance
- Micro-causality
- Primitive causality
- Primitivity
- Existence of the vacuum
- Weak additivity

Constructive field theory is the project of constructing models of the axioms and studying the behavior of those models.

(Selected) Achievements

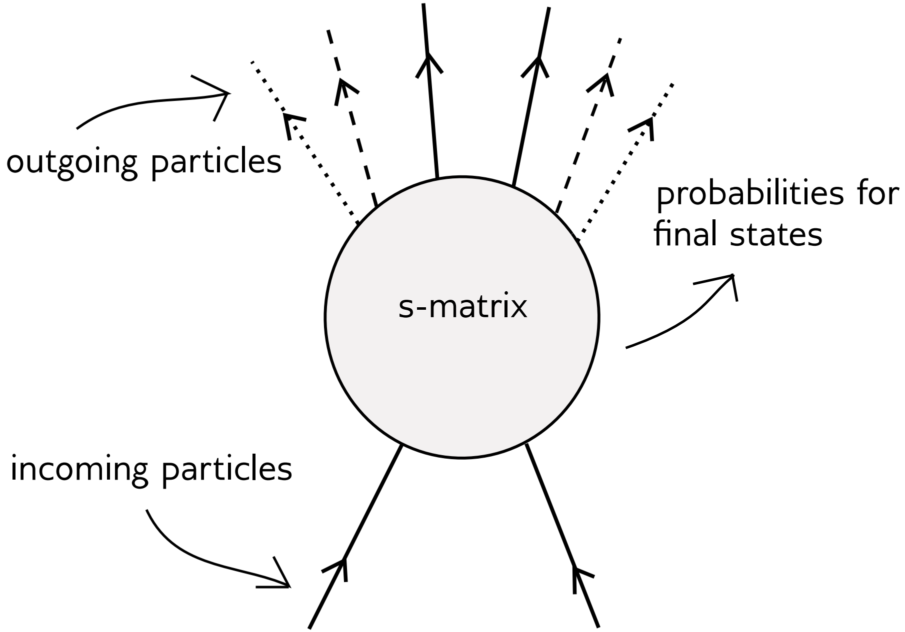
General theorems (CPT, spin-statistics)

Analysis of specific interpretive issues (particle concept, locality, quantization, ...)

Non-trivial models (but in reduced spacetime dimension, or without interactions)

A concern: empirically adequate field theories have not been shown to be models of the axioms. So should we expect any of this analysis to carry over to empirically adequate models?

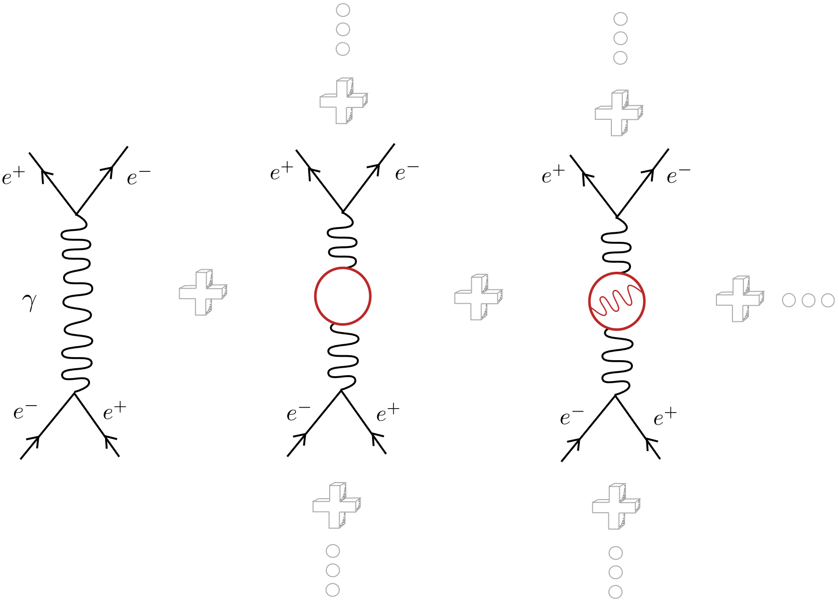
The perturbative approach: Treat the interacting field problem as a small perturbation of the well-defined free field problem.


outgoing particles
probabilities for
final states
s-matrix
incoming particles

numbers

powers of the coupling

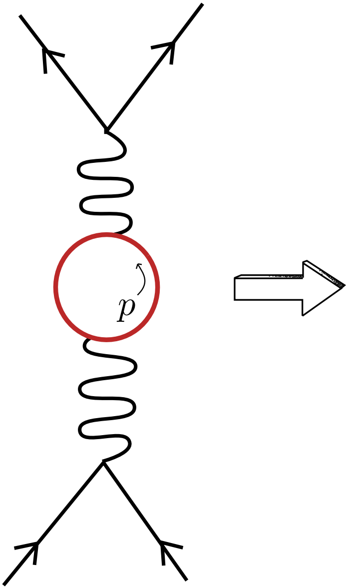
$$\text{s-matrix} \rightarrow \sum_{n=0}^{\infty} a_n \alpha^n$$



First order

Second order

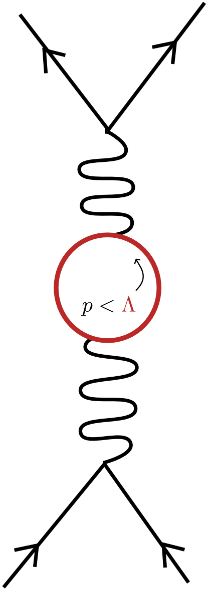
Third order



$$\int_0^\infty \frac{dp}{p} = \ln(\infty) = \infty$$

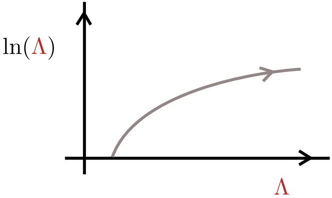
The theory predicts infinite probability?!?

$$O_{\text{th}} \notin O_{\text{ex}} \pm \epsilon_{\text{ex}}$$



$$\int_0^{\Lambda} \frac{dp}{p} = \ln(\Lambda) = \text{finite}$$

(for finite Λ)



There is a principled and physically well-motivated rationale behind the renormalization procedure.

Empirically adequate field theories are effective field theories.

The measurable low-momentum empirical content is insensitive to the high-momentum dynamics that we ignore.

This is a new view of how theories work which invites a revisitation of the connection between physics and metaphysics.

References

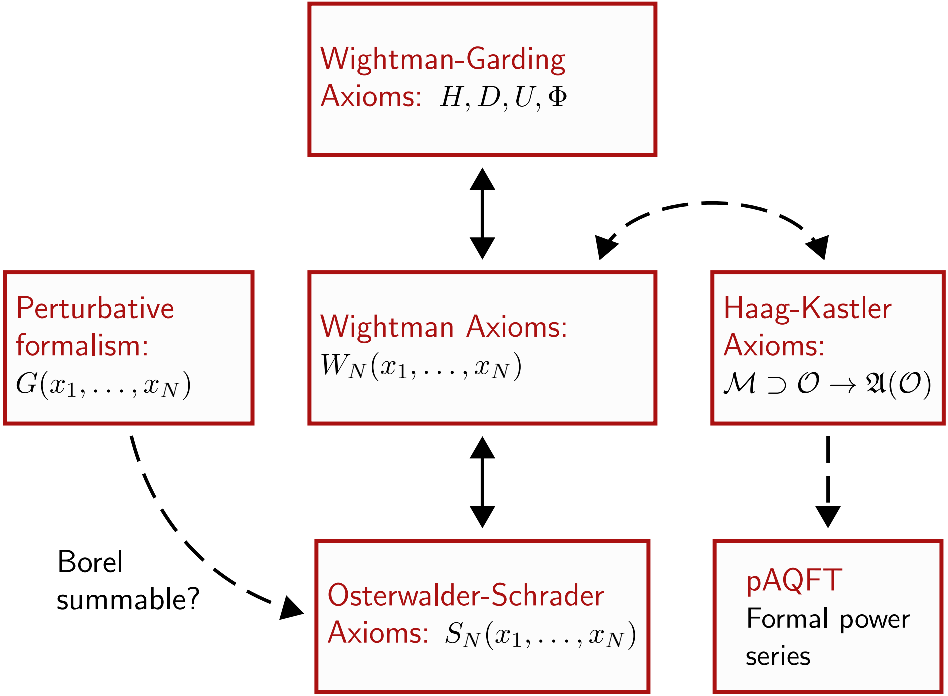
Fraser, D. (2011). How to take particle physics seriously: A further defence of axiomatic quantum field theory. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics 42*(2), 126–135.

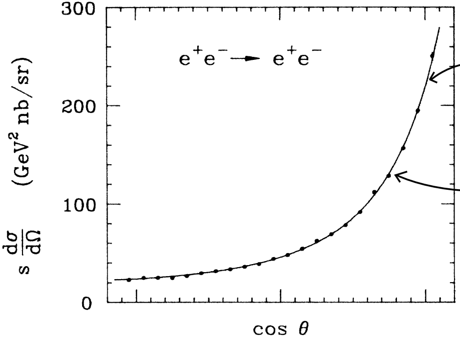
Ruetsche, L. (2011). *Interpreting Quantum Theories*. Oxford University Press.

Segal, I. E. (1959). The Mathematical Meaning of Operationalism in Quantum Mechanics. In L. Henkin, P. Suppes, and A. Tarski (Eds.), *The Axiomatic Method With Special Reference to Geometry and Physics*, pp. 341–352. Amsterdam: North Holland Publishing Company.

Wallace, D. (2011). Taking particle physics seriously: A critique of the algebraic approach to quantum field theory. *Studies in History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics 42*(2), 116–125.

Additional slides


Wightman-Garding Axioms: H, D, U, Φ
Perturbative formalism: $G(x_1, \ldots, x_N)$
Wightman Axioms: $W_N(x_1, \ldots, x_N)$
Haag-Kastler Axioms: $\mathcal{M} \supset \mathcal{O} \to \mathfrak{A}(\mathcal{O})$
Borel summable?
Osterwalder-Schrader Axioms: $S_N(x_1, \ldots, x_N)$
pAQFT
Formal power series



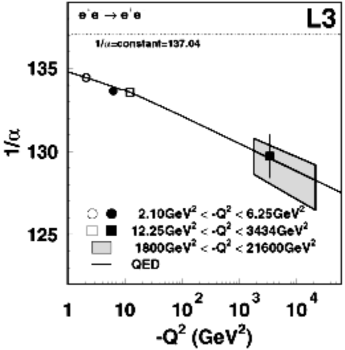
Sum of first three terms

Empirical data

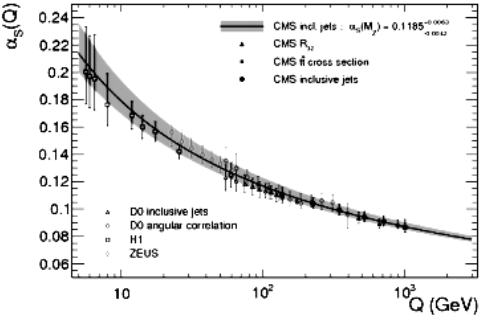
$O_{\text{th}} \in O_{\text{ex}} \pm \epsilon_{\text{ex}}$

M. Derrick et. al., Phys. Rev. **D34**, 3286 (1986).

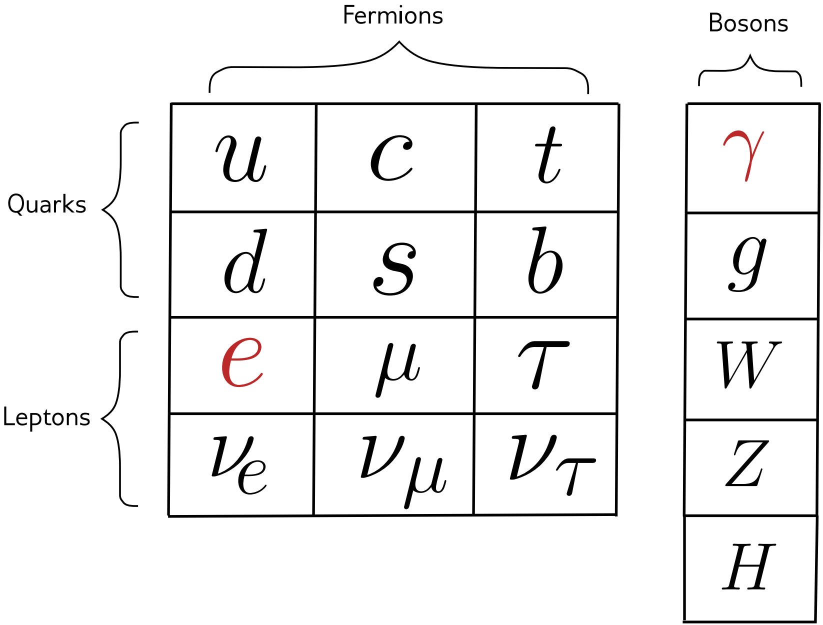
Running QED coupling



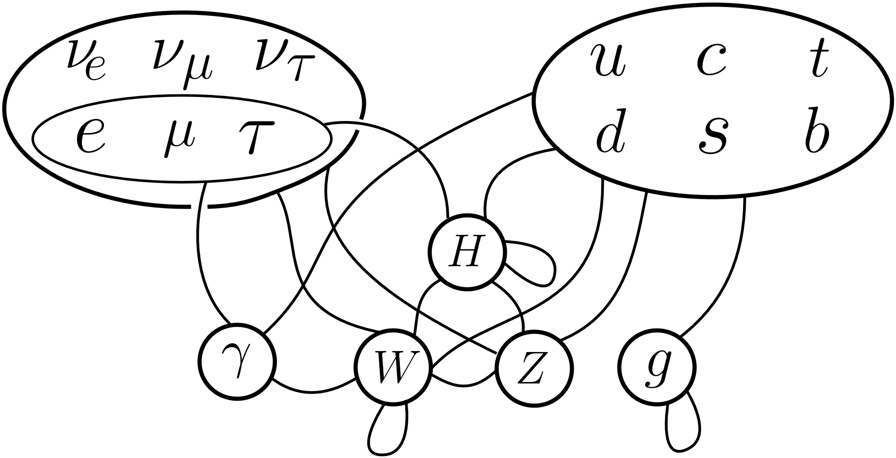
Running QCD coupling



(Left) LEP Collaboration. Phys. Lett. **B623** 26 (2005).
(Right) CMS Collaboration. Eur. Phys. J. **C75** 288 (2015).



Fermions
Bosons
Quarks
Leptons
u c t
d s b
e μ τ
ν_e ν_μ ν_τ
γ
g
W
Z
H


ν_e ν_μ ν_τ
e μ τ
u c t
d s b
H
γ
W
Z
g