Algorithmic Randomness and Probabilistic Laws

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Probabilistic Law L: Each element in the ω-sequence of coin tosses $\langle r_1, r_2, \ldots \rangle$ is determined independently and with an unbiased probability of heads and tails.

Empirical Coherence: A physical law is empirically coherent only if it is always in principle possible for one to have empirical support for the law if the law is in fact true.

Probabilistic Law $L^\star$: The ω-sequence of coin tosses $\langle r_1, r_2, \ldots \rangle$ is random with unbiased relative frequencies of heads and tails.

Kolmogorov-Chaitin Randomness: An ω-sequence is Kolmogorov-Chaitin random if and only if there is a constant c such that all finite initial segments are c-incompressible (by a prefix-free Turing machine).

Martin-Löf Randomness: An ω-sequence is Martin-Löf random if and only if it belongs to every effective full-measure set, i.e. it belongs to no effective measure-zero set.

Schnorr's Theorem: An ω-sequence is Martin-Löf random if and only if it is Kolmogorov-Chaitin random.

Probabilistic Law $L^\star_{ML}$: The ω-sequence of coin tosses $\langle r_1, r_2, \ldots \rangle$ is Martin-Löf random with unbiased relative frequencies of heads and tails.

Probabilistic Law $L^\star_S$: The ω-sequence of coin tosses $\langle r_1, r_2, \ldots \rangle$ is Schnorr random with unbiased relative frequencies of heads and tails.

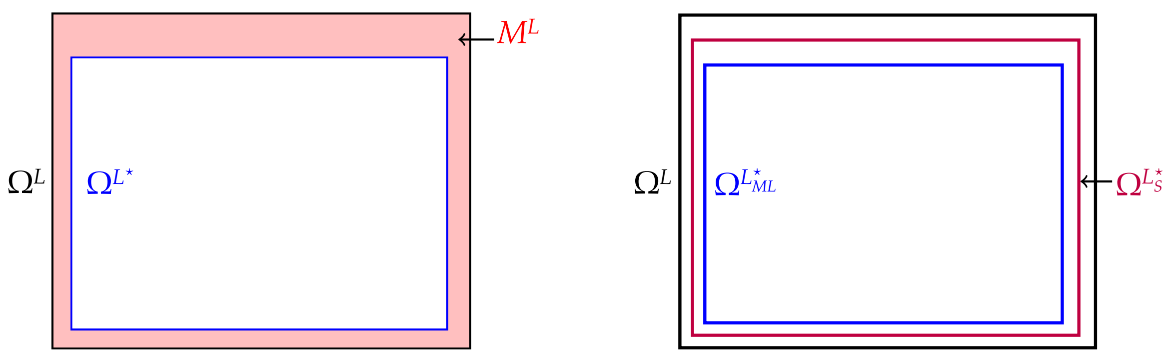

Figure 1: Ω^L: the set of worlds that accord with law L. M^L: maverick worlds in Ω^L.

$L^\star_{ML}$ and $L^\star_S$ are different laws. But they are in a strong sense empirically equivalent. No effective procedure would determine whether a particular sequence is Martin-Löf random or Schnorr random but not Martin-Löf random. (Barrett and Huttegger 2021)

We shall understand $L^\star$ as $L^\star_{ML}$.

One might think of $L^\star$ in one of these two ways:

- a generative chance* law;
- a probabilistic* constraining law.

As a generative chance* law:

- $L^\star$ tells us that each toss is generated by unbiased chances*,
- where a chance* process behaves just like an ordinary chance process except that it can never produce an infinite sequence that fails to be Martin-Löf random or fails to exhibit well-defined relative frequencies.
- Subtle violation of independence.

As a probabilistic* constraining law:

- $L^\star$ governs by constraining the entire history of the world—in this case, the full ω-sequence of outcomes.
- Which sequences of outcomes are physically possible? Those that satisfy the frequency constraint and the randomness constraint imposed by the law.
- cf. Chen and Goldstein's (2022) minimal primitivism account (MinP), according to which laws are certain primitive facts that govern the world by constraining the physical possibilities of the entire spacetime and its contents.
- Thinking of $L^\star$ this way yields a unified account of how laws govern.

Humeans may also find it useful to adopt $L^\star$, for two reasons:

- It eliminates the Big Bad Bug: undermining histories are physically impossible.
- It bypasses the problems of fit: we no longer need *fit* to break ties in the best system competition.

Summary:

- We have used algorithmic randomness to characterize two types of probabilistic laws: a generative chance* law and a probabilistic* constraining law.
- $\star$-laws provide a novel way of understanding probabilities and chances, and help to address one variety of empirical underdetermination, but they also reveal other varieties that have been underappreciated.
- For all we know, our world might be characterized by a traditional probabilistic law or a $\star$-law.