STRUCTURALISM AND ONTOLOGY

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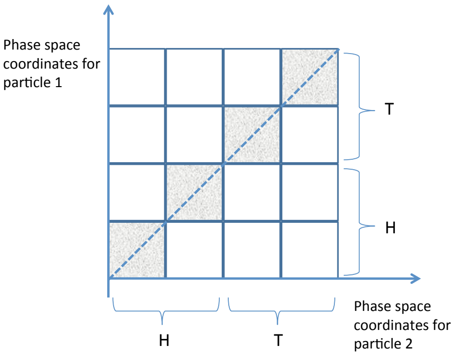
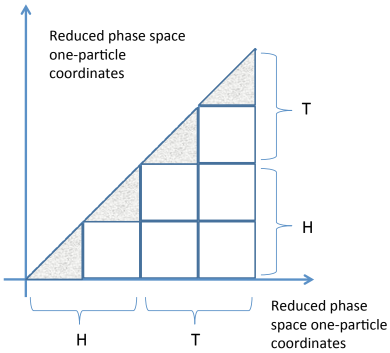
If two phases differ only in that certain entirely similar particles have changed places with one another, are they to be regarded as identical or different phases? If the particles are regarded as indistinguishable, it seems in accordance with the spirit of the statistical method to regard the phases as identical. In fact, it might be urged that …if the particles of one system are described as entirely similar to one another and to those of another system, nothing remains on which to base the identification of any particular particle of the first system with any particular particle of the second. And this would be true, if the ensemble of systems had a simultaneous objective existence. But it hardly applies to the creations of the imagination.
(*Principles of Statistical Mechanics, 1902*. Ch. 15)

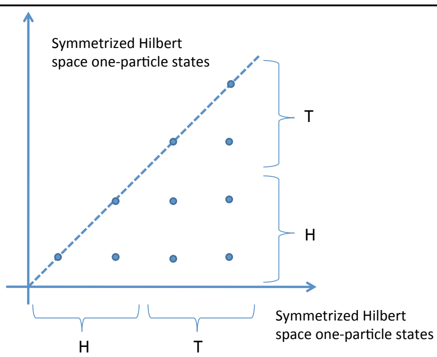
J. W. Gibbs 1839-1903



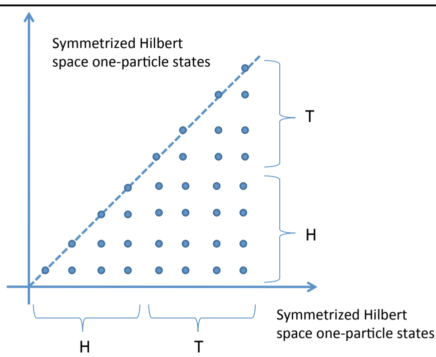
Shaded cells have no partners on particle interchange



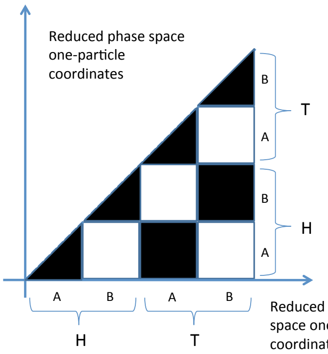
Shaded cells have half the volume of cells off the diagonal



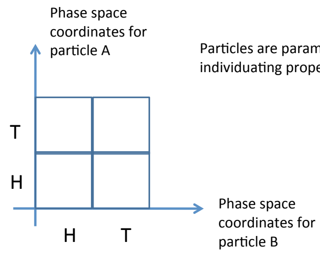
Discrete measure for symmetrized Hilbert space



As the number of cells increases, the discrete measure becomes approximately proportional to dimension of unsymmetrized subspace



Shaded areas represent dynamically inaccessible regions of phase space



Particles are parameterized by individuating properties A and B

In the case of 'trivial' entanglements, can individuate by one-particle states:

$$|\Phi\rangle = c \sum_{\pi \in \Pi_N} |\phi_{\pi(a)}\rangle \otimes |\phi_{\pi(b)}\rangle \otimes \ldots \otimes |\phi_{\pi(c)}\rangle \otimes \ldots \otimes |\phi_{\pi(d)}\rangle$$

$$|\Psi^{FD}\rangle = \frac{1}{\sqrt{N!}} \sum_{\pi \in \Pi_N} \mathrm{sgn}(\pi) |\phi_{\pi(a)}\rangle \otimes |\phi_{\pi(b)}\rangle \otimes \ldots \otimes |\phi_{\pi(c)}\rangle \otimes \ldots \otimes |\phi_{\pi(d)}\rangle$$

See Ghirardi and Marinatto (2002, 2004) for 'triviality' claim.

See also Goldstein, Taylor, Tumulka, and Zanghi (2005) for claim that can always (anti)symmetrize.

Symmetrized quantum 1-particle property (symmetrized projection):

$P\otimes(1-P) \otimes \ldots\ldots \otimes(I-P)$ +

$(1-P)\otimes P\otimes(1-P) \otimes \ldots\ldots\ldots \otimes(1-P)$ +....

+ $(1-P)\otimes \ldots\ldots\ldots \otimes(1-P)\otimes P$

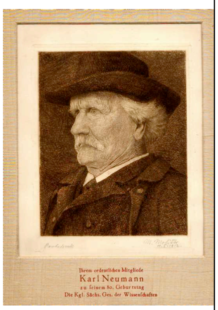
Symmetrized predicate:

$(Pa_1 \,\&\neg Pa_2 \,\&\ldots\ldots\&\neg Pa_N) \vee$

$(\neg Pa_1 \,\&\, Pa_2 \,\&\neg Pa_3 \,\&\ldots\ldots\&\neg Pa_N) \vee \ldots$

$\vee (\neg Pa_1 \,\&\ldots\&\neg Pa_{N-1} \,\&\, Pa_N)$

In a purely mathematical investigation involving several variables simultaneously in which the relationship between these variables is to be represented in as clear a manner as possible, it is often expedient or even necessary to introduce an intermediate variable and then specify the relationship which each of the given variables has to this intermediate quantity. We find something similar in the physical theories. In order to get an overview of the connection between different phenomena presented simultaneously, it is often expedient to introduce a merely conceptual process, a merely conceptual substance, that, so to speak, represents an intermediate principle, a central point, from which the individual phenomena can be reached in different directions. The individual phenomena are linked to each other in this manner, in that each is related to the central point. Such is the role played by the luminous ether in the theory of optical phenomena, and the electric fluid in the theory of electric phenomena; and our Body Alpha plays a similar role in the general theory of motion. (Inaugural lecture 1870)



C. Neumann 1832-1925

Vector-space relationalism

Go from Newton's laws

$$m_i \ddot{x}_i = \sum_{k \neq i} F(x_i - x_k)$$

To the (1/2 (n-1) linearly independent) equations for the relative accelerations

$$\ddot{x}_i - \ddot{x}_j = \frac{1}{m_i} \sum_k F(x_k - x_i) - \frac{1}{m_j} \sum_k F(x_j - x_k)$$

With symmetries

$$x_i \to R x_i + f(t)$$

"letting go" of the equation for the centre of mass:

$$x_{com} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

But then as David Wallace has observed:

$$\ddot{x}_i - \ddot{x}_{com} = \frac{1}{m_i} \sum_{k \neq i} F(x_i - x_k)$$

(and likewise for coordinates related by rotations, translations and boosts) and we recover the Galilean group – Galilean spacetime is *emergent* (Wallace)

Quantum histories formalism:

"Building blocks of reality" (von Neumann, Everett), projectors onto configurations α_k

$$P_{\alpha_k}, \quad \sum_{\alpha_k} P_{\alpha_k} = \mathbb{1}$$

Index them to times (Heisenberg picture):

$$P_{\alpha_k}(t_k) = e^{iHt_k} P_{\alpha_k} e^{-iHt_k}$$

Given a sequence $\underline{\alpha} = \alpha_n \dots \alpha_1$ define *histories*

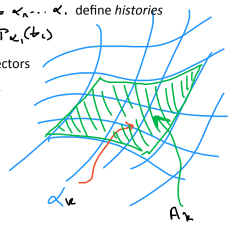
$$C_{\underline{\alpha}} = P_{\alpha_n}(t_n) \dots P_{\alpha_1}(t_1)$$

Can coarse grain projectors

$$P_{A_k} = \sum_{\alpha_k \in A_k} P_{\alpha_k}$$

And likewise histories:

$$C_{\underline{A}} = \sum_{\underline{\alpha} \in \underline{A}} C_{\underline{\alpha}}$$



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Define composition of configurations by product of associated (commuting) projectors:

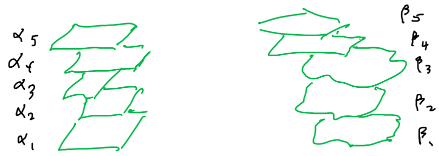
$$P_{\alpha_k * \beta_k}(t_k) = P_{\alpha_k}(t_k) P_{\beta_k}(t_k)$$

Think of (temporarily) as of the form:

$$P_{\alpha_k} = P_{\alpha_k} \otimes 1 \otimes 1, \quad P_{\beta_k} = 1 \otimes 1 \otimes P_{\beta_k}$$

Extend to histories in the obvious way:

$$C_{\underline{\alpha} * \underline{\beta}} = P_{\alpha_n * \beta_n}(t_n) \dots P_{\alpha_1 * \beta_1}(t_1)$$



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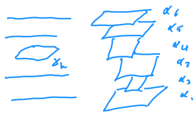
Call histories $\underline{\alpha}$, $\underline{\beta}$ *compossible* if $|C_{\underline{\alpha} * \underline{\beta}}| \neq 0$

If *decoherent*, can define probabilities of histories relative to one another:

$$\mu(\underline{\alpha} / \underline{\beta}) = \frac{\mathrm{Tr}\left[C^{\dagger}_{\underline{\alpha} * \underline{\beta}} \, \rho \, C_{\underline{\alpha} * \underline{\beta}}\right]}{\mathrm{Tr}\left[C^{\dagger}_{\underline{\beta}} \, \rho \, C_{\underline{\beta}}\right]}$$

Can likewise consider probability of an event γ_k relative to a history (embed the event in the trivial history)

Generalises the two-vector formalism of Aharonov et al.



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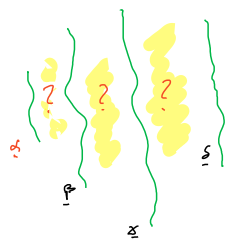
Quantum monadology?



Without a tensor product structure, there can be no two identical monads, as:

$$\begin{aligned} C_{\underline{\alpha} * \underline{\alpha}} &= P_{\alpha_1 * \alpha_1}(t_1) \dots P_{\alpha_n * \alpha_n}(t_n) \\ &= P_{\alpha_1}(t_1) P_{\alpha_1}(t_1) \dots P_{\alpha_n}(t_n) P_{\alpha_n}(t_n) \\ &= P_{\alpha_1}(t_1) \dots P_{\alpha_n}(t_n) = C_{\underline{\alpha}} \end{aligned}$$

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