Bohmian quantum field theory

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• Non-relativistic Bohmian mechanics
  – Point-particles with positions $X_1, \ldots, X_n$
  – Dynamics depends on the wave function $\psi$
    \[
    \frac{dX_i}{dt} = v_i(\mathbf{X}_1, \ldots, \mathbf{X}_n)
    \]
  – $\psi$ satisfies non-relativistic Schrödinger equation
    \[
    i\partial_t \psi = H_{n.r.}\psi
    \]

• Bohmian quantum field theory
  – Point-particles or fields?
  – Dynamics that depends on $\psi$
  – $\psi$ satisfies Schrödinger equation
    \[
    i\partial_t \psi = H_{QFT}\psi
    \]
Non-relativistic Bohmian mechanics
(a.k.a. pilot-wave theory, de Broglie-Bohm theory, ...)

- De Broglie (1927), Bohm (1952)
- Point-particles moving under influence of the wave function.
- Positions of the particles: $X_1, \ldots, X_n$
  
  Configuration $X = (X_1, \ldots, X_n)$
Dynamics:

$$\frac{dX(t)}{dt} = v^\psi_t(X(t))$$

where

$$v^\psi = (v_1^\psi, \ldots, v_n^\psi), \quad v_k^\psi = \frac{\hbar}{m_k} \text{Im} \frac{\nabla_k \psi}{\psi} = \frac{1}{m_k} \nabla_k S, \quad \psi = |\psi| e^{iS/\hbar}$$

$$i\hbar \partial_t \psi_t(x) = \left( -\sum_{k=1}^N \frac{\hbar^2}{2m_k} \nabla_k^2 + V(x) \right) \psi_t(x), \quad x = (x_1, \ldots, x_N)$$
• Double Slit experiment:
Quantum equilibrium and standard quantum predictions

- Quantum equilibrium:
  - for an ensemble of systems with wave function $\psi$
  - equilibrium distribution of particle positions $\rho(x) = |\psi(x)|^2$

- Quantum equilibrium is preserved by the particle motion (≡ equivariance), i.e.

  $$\rho(x, t_0) = |\psi(x, t_0)|^2 \quad \Rightarrow \quad \rho(x, t) = |\psi(x, t)|^2 \quad \forall t$$

because guidance equation implies

$$\partial_t \rho + \sum_{k=1}^{n} \nabla_k \cdot (v_k^\psi \rho) = 0$$

and Schrödinger equation implies

$$\partial_t |\psi|^2 + \sum_{k=1}^{n} \nabla_k \cdot (v_k^\psi |\psi|^2) = 0$$

- Agreement with quantum theory in quantum equilibrium. We have quantum equilibrium in a typical universe (Dürr, Goldstein, Zanghì).
Recipe to construct Bohmian theories for a given quantum theory

- Choose ontology in space(-time), also called local beables, say particles $X$
- Consider the quantum distribution $|\psi|^2$
- Consider the continuity equation for $|\psi|^2$:
  \[
  \partial_t |\psi|^2 + \text{div} \dot{j}_\psi = 0
  \]
- Define guidance equation
  \[
  \dot{X} = \frac{j_\psi}{|\psi|^2}
  \]

This recipe will guarantee equivariance of the equilibrium distribution.
But does not necessarily guarantee an empirically adequate theory:

The variables $X$ should contain, on the macroscopic level, an image of the everyday classical world (Bell)
Effective collapse of the wave function

- Branching of the wave function: \( \psi \rightarrow \psi_1 + \psi_2 \) \( \psi_1 \psi_2 = 0 \)
- Effective collapse \( \psi \rightarrow \psi_1 \) (\( \psi_2 \) does no longer effect the motion of the configuration \( X \))
• Stern-Gerlach experiment

Sketch of the setup:

- Inhomogeneous magnetic field in the $z$-direction
- SOURCE
- Produces particles with spin up in the $x$-direction
- Radiation scattering of the needle
- Measurement needle
- Detection screen
Orthodox quantum description:

\[ |+\rangle_x |\text{up} \rangle | \rightarrow | \text{up} \rangle \]

(Schrödinger evolution)

\[ \rightarrow \frac{1}{\sqrt{2}} |+\rangle_z |\text{up} \rangle | \rightarrow | \text{up} \rangle + \frac{1}{\sqrt{2}} |-\rangle_z |\text{up} \rangle | \rightarrow | \text{up} \rangle \]

(Collapse)

\[ \rightarrow |+\rangle_z |\text{up} \rangle | \rightarrow | \text{up} \rangle \]
Bohmian treatment of spin

– No need to introduce extra variables representing spin.

Just positions:

\[
\frac{dX_k}{dt} = \frac{\hbar}{m_k} \text{Im} \frac{\psi^\dagger \nabla_k \psi}{\psi^\dagger \psi},
\]

(1)

Guidance law for non-relativistic spin-1/2 particles:

where \( \psi \) is Pauli spinor.
Bohmian picture:

AT TIME $t_1$

AT TIME $t_2 > t_1$
One could introduce extra variables. E.g. the spin vector:

\[ s(t) = \frac{\hbar \psi^\dagger(x, t) \sigma \psi(x, t)}{2 \psi^\dagger(x, t) \psi(x, t)} \bigg|_{x=X(t)} \]

which is constructed from the Pauli spinor \( \psi \) and the actual position of the particle.
• Dirac theory (first-quantized level)

- Single particle
  
  Dirac equation:
  \[ i\partial_\mu \gamma^\mu \psi - m\psi = 0 \]

  Guidance equation:
  \[ \frac{dX^\mu}{d\tau} \sim j^\mu = \psi^\dagger \gamma^0 \gamma^\mu \psi \]

  \[ \rightarrow \text{Is fully Lorentz invariant} \]
– Many particles

Dynamics depends on particular reference frame or foliation $\mathcal{F}$ (to guarantee equivariance):

$$\dot{X}_k(\Sigma) = \psi_k \mathcal{F}(X_1(\Sigma), \ldots, X_N(\Sigma))$$

This dependence makes the theory not Lorentz invariant. Nevertheless the preferred frame or foliation cannot be detected. The statistical predictions are Lorentz invariant.

Dynamics can be made Lorentz invariant by a covariant determination of the foliation by the wave function: $\mathcal{F} = \mathcal{F}^\psi$ (Dürr, Goldstein, Norsen, Struyve, Zanghì 2014)
Bohmian quantum field theory

- Need suitable variables (local beables). Particles, fields?
- Need wave function and Schrödinger evolution.
- We assume suitable regulators which make equations well-defined.
- We assume preferred reference frame.
**Field ontology**

**Bosonic fields**

- **Scalar field** (Bohm (1952))

  Hamiltonian:
  \[
  \hat{H} = \frac{1}{2} \int d^3x \left( \hat{\Pi}^2 + (\nabla \hat{\phi})^2 + m^2 \hat{\phi}^2 \right), \quad [\hat{\phi}(x), \hat{\Pi}(y)] = i\delta(x - y)
  \]

  Functional Schrödinger representation:
  \[
  \hat{\phi}(x) \rightarrow \phi(x), \quad \hat{\pi}(x) \rightarrow -i\frac{\delta}{\delta \phi(x)}
  \]

  \[
  i\frac{\partial \Psi(\phi, t)}{\partial t} = \frac{1}{2} \int d^3x \left( -\frac{\delta^2}{\delta \phi^2} + (\nabla \phi)^2 + m^2 \phi^2 \right) \Psi(\phi, t).
  \]

  Bohmian field \(\phi(x)\) with guidance equation:
  \[
  \frac{\partial \phi(x, t)}{\partial t} = \frac{\delta S(\phi, t)}{\delta \phi(x)} \bigg|_{\phi=\phi(x,t)}\, \Psi = |\Psi|e^{iS}
  \]

- **Similarly for other bosonic fields** (see Struyve (2010) for a review).

  For example, electromagnetic field: \((A(x), \Psi(A))\)
Fermionic fields

• Valentini (1992)
  – Wave function $\Psi(\eta)$,
    $\eta(x)$ is a Grassmannian field,
    $\Psi$ is an element of a Grassmann algebra ($\text{not } \mathbb{C}$)
  – Local beable: $\eta(x)$
  – Problem: What is the equilibrium distribution? What is an adequate guidance equation?
• Holland (1988, 1993)
  
  - Wave function $\Psi(\alpha)$,
    
    $$\alpha(k) = (\alpha(k), \beta(k), \gamma(k))$$
    
    set of Euler angles for each point in momentum space
Fermionic fields

• Holland (1988,1993)
  - Wave function $\Psi(\alpha)$,
    $\alpha(k) = (\alpha(k), \beta(k), \gamma(k))$ set of Euler angles for each point in momentum space
  - Local beable: Isn’t specified by Holland, but could be constructed from beables $\alpha(k)$ and the wave function.
    - Hard to see whether it is empirically adequate

• Variant of Holland’s approach, Struyve (2007)
  - Wave function $\Psi(\alpha)$,
    $\alpha(x) = (\alpha(x), \beta(x), \gamma(x))$ set of Euler angles for each point in physical space
  - Local beable: $\alpha(x)$
    - Seems empirically inadequate!
Stern-Gerlach experiment:

TIME $t_a$

TIME $t_a > t_b$
Minimalist model

  - Wave function: \( \Psi_f(A) = \langle f, A | \Psi \rangle \),
    \( f \) labels the fermionic degrees of freedom
    \( A(x) \) is the vector potential
  - Local beable: \( A(x) \)
    None for fermionic degrees of freedom
  - Equilibrium distribution: \( \sum_f |\Psi_f(A)|^2 \)
Stern-Gerlach experiment:

TIME $t_1$

\[
\begin{align*}
\left( \left( \right) \right) & \quad = \quad \sim
\end{align*}
\]

TIME $t_2 > t_1$

\[
\begin{align*}
\left( \left( \right) \right) & \quad \sim
\end{align*}
\]
Minimalist model + extras

- Struyve & Westman (2007)
  - Wave function: $\Psi_f(A) = \langle f, A | \Psi \rangle$,
  - Local beables:
    - potential $A(x)$
    - charge distribution $\rho(x)$, given by
      \[
      \rho(x, t) = \frac{\sum_{f,f'} \Psi_f^*(A, t) \hat{\rho}_{ff'}(x) \Psi_{f'}(A, t)}{\sum_f |\Psi_f(A, t)|^2} \bigg|_{A=A(t)}
      \]
      where $\hat{\rho}_{ff'}(x) = \langle f | \hat{\rho}(x) | f' \rangle$, with $\hat{\rho}(x)$ the charge density operator
Stern-Gerlach experiment:

TIME $t_1$

TIME $t_2 > t_1$
Particle ontology

- Bell (1984)
  - Considers spatial lattice
  - Local beables: fermion numbers $n = (n_1, \ldots, n_l)$ at the lattice points
  - Stochastic dynamics:
    Transition rates to jump from configuration $m$ to configuration $n$:

\[
T_{nm} = \frac{J_{nm}^+}{P_m}
\]

where $J^+ = \max(J, 0)$, with

\[
J_{nm} = \sum_{qp} 2\text{Re}\langle\psi(t)|nq\rangle\langle nq| - iH|mp\rangle\langle mp|\psi(t)\rangle
\]

and

\[
P_m = \sum_q |\langle mq|\psi(t)|^2
\]
• Continuum generalizations of Bell’s model:

   → Local beables: Particle positions
   → Explicit particle creation and annihilation
   → Stochastic model: There is deterministic motion interrupted by particle creation and annihilation which are stochastic

   → Local beables: Particle positions
   → Dirac sea picture (number of particles does not change)
   → Deterministic
Pair creation in the standard picture:
Pair creation in the Dirac sea picture:
Stern-Gerlach experiment in the Dirac sea - pilot-wave picture:
Conclusion

• In Bohmian QFT:
  – particles positions seem natural for fermions
  – fields seem natural for bosons

• Two candidates for fermions:
  – stochastic one based on the particle–anti-particle picture of QFT
  – deterministic one based on the Dirac sea picture

• The Bohmian theories reproduce the predictions of regularized quantum field theory. The regularization could be understood as a way of dealing with as yet unknown physics at high energies.

Perhaps the regularization can be removed while preserving a well-defined dynamics for the local beables (even though the wave function evolution might no longer be well-defined).